System $F$ with exceptions

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1 Introduction

2 Presenting $F_x$
   - The terms side
   - The types side

3 A realisability model of $F_x$

4 Conclusion and perspectives
What are exceptions?

Exceptions are a convenient mechanism to handle “errors” or “exceptional behavior”.
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  - Mostly associated to call-by-value reduction.
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- It is used in many (most) programming languages: OCaml, Java, Ruby, ... .
- In functional languages:
  - Mostly associated to call-by-value reduction.
  - Generally not precisely typed.
- What about exceptions in type theoretical settings?
An exception usually interrupts all computations until reaching an appropriate handler:
Exceptions propagation

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$$(\text{raise } E) A \triangleright \text{raise } E$$  \hspace{1cm} (raise_{cbn})

We do not want to keep the rule $$(\text{raise } cbv)$$ because:

- it is more related to call-by-value $\beta$ evaluation.
- Makes less sense with call-by-name.
- with $$(\text{raise } cbn)$$, not confluent.
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- $F (\text{raise } E) \triangleright \text{raise } E$ \hspace{1cm} (raise$_{cbv}$)
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An exception usually interrupts all computations until reaching an appropriate handler:

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- $F \ (\text{raise } E) \Rightarrow \text{raise } E \quad (\text{raise}_{\text{cbv}})$

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- with \(\text{raise}_{\text{cbn}}\), not confluent.
Exceptions as values

Exceptions are not anymore modifications of the control flow, but rather a special kind of values.
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In particular, some new values appear. For example, if we use primitive natural numbers with 0 and S, “S (raise E)” is not reducible.
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In particular, some new values appear. For example, if we use primitive natural numbers with 0 and $S$, “$S \text{ (raise } E\text{)}$” is not reducible.

And we want to be able to type such a value.
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What is $Fx$?

An extension of System $F\eta$ with typed exceptions and primitive natural numbers.
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What is $F_X$?

An extension of System $F_\eta$ with typed exceptions and primitive natural numbers.

- A subtyping relation noted $\leq$.
- With raise and try constructions.
- With new type constructions to type it.
- All the typing and subtyping rules of $F_\eta$ are unchanged. We only add rules.
What is $F_x$?

An extension of System $F\eta$ with typed exceptions and primitive natural numbers.

- A subtyping relation noted $\leq$.
- With `raise` and `try` constructions.
- With new type constructions to type it.
- All the typing and subtyping rules of $F\eta$ are unchanged. We only add rules.
- Justified by a realisability model.
$\mathcal{E}$ : a countable set of names of exceptions.
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The syntax of $Fx$ is defined by:

\[
M, N ::= \lambda x. M | \varepsilon | \text{try } M \text{ with } \varepsilon \mapsto N
\]
\( E \) : a countable set of names of exceptions.

The syntax of \( Fx \) is defined by:

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M, N ::= x \mid \lambda x. M \mid M N
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$\mathcal{E}$ : a countable set of names of exceptions.

The syntax of $Fx$ is defined by:

$$M, N ::= x \mid \lambda x. M \mid M \; N \mid \text{raise} \; \varepsilon \mid \text{try} \; M \; \text{with} \; \varepsilon \; \mapsto \; N$$

where $\varepsilon \in \mathcal{E}$.
$E$: a countable set of names of exceptions.

The syntax of $Fx$ is defined by:

$$M, N ::= x \mid \lambda x. M \mid M N \mid \text{raise } \varepsilon \mid \text{try } M \text{ with } \varepsilon \mapsto N \mid 0 \mid S \mid \text{Rec} \mid \text{eval}$$

where $\varepsilon \in E$. 
\( \mathcal{E} \) : a countable set of names of exceptions.

The syntax of \( Fx \) is defined by:

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M, N ::= \ x \mid \lambda x. M \mid M \ N \mid \text{raise } \varepsilon \mid \text{try } M \text{ with } \varepsilon \mapsto N \\
\mid 0 \mid S \mid \text{Rec} \mid \text{eval}
\]

where \( \varepsilon \in \mathcal{E} \).

We also define a notion of value:

\[
V ::= \lambda x. M \mid 0 \mid S \mid S \ N \mid \text{Rec} \mid \text{Rec } M \mid \text{Rec } M \ N \mid \text{eval}
\]
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Reduction in $Fx$ (1/2)

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\begin{align*}
(\lambda x. M) N & \quad \Rightarrow \quad M\{x := N\} \\
(\text{raise } \varepsilon) M & \quad \Rightarrow \quad \text{raise } \varepsilon \\
\text{try}(\text{raise } \varepsilon)\text{ with } \varepsilon \mapsto N & \quad \Rightarrow \quad N \\
\text{try}(\text{raise } \varepsilon')\text{ with } \varepsilon \mapsto N & \quad \Rightarrow \quad \text{raise } \varepsilon' \\
\text{try } V\text{ with } \varepsilon \mapsto N & \quad \Rightarrow \quad V
\end{align*}
\]
The notion of reduction is defined by the following rules:

- $(\lambda x. M) \ N \rightarrow M\{x := N\}$
- $(\text{raise } \varepsilon) \ M \rightarrow \text{raise } \varepsilon$
- $\text{try}(\text{raise } \varepsilon) \text{ with } \varepsilon \mapsto N \rightarrow N$
- $\text{try}(\text{raise } \varepsilon') \text{ with } \varepsilon \mapsto N \rightarrow \text{raise } \varepsilon'$
- $\text{try } V \text{ with } \varepsilon \mapsto N \rightarrow V$
- $\text{Rec } X \ Y \ 0 \rightarrow X$
- $\text{Rec } X \ Y \ (S \ N) \rightarrow Y \ N \ (\text{Rec } X \ Y \ N)$
- $\text{Rec } X \ Y \ (\text{raise } \varepsilon) \rightarrow \text{raise } \varepsilon$
eval : “to bring exceptions up”
Reduction in $\mathcal{Fx}$ (2/2)

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- eval $S (S (S (S (\text{raise } \varepsilon)))) > \text{raise } \varepsilon$
Reduction in $F_x$ (2/2)

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- $\text{eval } S (S (S (S (\text{raise } \varepsilon)))) > \text{raise } \varepsilon$
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Reduction in $Fx$ (2/2)

eval : “to bring exceptions up”

- eval $S (S (S (S (raise \varepsilon))))$ > raise $\varepsilon$
- eval $S (S (S (S 0)))$ > $S (S (S (S 0)))$

Easily defined with Rec and by using an accumulator.
Types of $F_x$

The syntax for the types of $F_x$ is given by:

$$A, B ::=$$
Types of $F_x$

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$$A, B ::= \alpha | \mathbb{N} | A \rightarrow B | \forall \alpha. A$$
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$$A, B ::= \alpha | \mathbb{N} | A \rightarrow B | \forall \alpha. A$$

$$| A \cup \Delta \quad \text{(terms from } A \text{ or exceptions of } \Delta)$$

Où $\Delta \subseteq \mathcal{E}$
Types of $Fx$

The syntax for the types of $Fx$ is given by:

$$A, B ::= \alpha | \mathbb{N} | A \rightarrow B | \forall \alpha. A$$

$$\quad | A \uplus \Delta \quad \text{(terms from } A \text{ or exceptions of } \Delta)$$

$$\quad | A^\Delta \quad \text{(terms corrupted by exceptions of } \Delta)$$

Où $\Delta \subseteq \mathcal{E}$
$A \uplus \Delta$ : terms of type $A$ or exceptions of names in $\Delta$. 
\( A \ast \Delta \): terms of type \( A \) or exceptions of names in \( \Delta \).

- 0: \( \mathbb{N} \ast \{\varepsilon\} \)
\[ A \uplus \Delta : \text{terms of type } A \text{ or exceptions of names in } \Delta. \]

- \[ 0 : \mathbb{N} \uplus \{ \varepsilon \} \]
- \[ \text{raise}\varepsilon : \mathbb{N} \uplus \{ \varepsilon \} \]
\( A \uplus \Delta \): terms of type \( A \) or exceptions of names in \( \Delta \).

- \( 0 : \mathbb{N} \uplus \{ \varepsilon \} \)
- \( \text{raise} \varepsilon : \mathbb{N} \uplus \{ \varepsilon \} \)

Terms of \( A \uplus \Delta \) are the ones caught by a try.
but it has its limitations

We still have some problems unsolved.
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What type to give to $S \ (\text{raise } \varepsilon)$?
but it has its limitations

We still have some problems unsolved.

What type to give to $S\ (\text{raise } \epsilon)$?
The type $\mathbb{N} \cup \{\epsilon\}$ is of course not appropriate.
but it has its limitations

What about \((A \rightarrow B) \cup \Delta\)?
but it has its limitations

What about $(A \to B) \uplus \Delta$?

$$f : (A \to B) \uplus \{\varepsilon\}$$
but it has its limitations

What about \((A \rightarrow B) \uplus \Delta\)?

\[
f : (A \rightarrow B) \uplus \{\varepsilon\}
\]

\[
f : A \rightarrow B
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but it has its limitations

What about \((A \rightarrow B) \uplus \Delta\)?

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f : (A \rightarrow B) \uplus \{\varepsilon\}
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\[
f : A \rightarrow B \\
\]

\[
f > \text{raise } \varepsilon
\]
but it has its limitations

What about \((A \rightarrow B) \cup \Delta\)?

\[
f : (A \rightarrow B) \cup \{\varepsilon\}
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\[
f : A \rightarrow B \quad f > \text{raise} \varepsilon
\]

But \(f\) is not allowed by typing to be applied to a term of \(A\).
but it has its limitations

What about \((A \rightarrow B) \uplus \Delta\)?

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f : (A \rightarrow B) \uplus \{\varepsilon\}
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\[
f : A \rightarrow B
\]

\[
f > \text{raise } \varepsilon
\]

But \(f\) is not allowed by typing to be applied to a term of \(A\)

Type construction \(A \uplus \Delta\) doesn’t deal well with arrow types.
And so comes the corruption

A term of $A^\Delta$ is a term of $A$ for which some subterms could have been replaced by an exception.
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Another way to see it,
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Another way to see it,

- a term of $A \uplus \Delta$ may raise an exception at top-level.
And so comes the corruption

A term of $A^\Delta$ is a term of $A$ for which some subterms could have been replaced by an exception.

Another way to see it,

- a term of $A \cup \Delta$ may raise an exception at top-level.
- a term of $A^\Delta$ may raise an exception in some evaluation context (of $A$).
And so comes the corruption

A term of $A^\Delta$ is a term of $A$ for which some subterms could have been replaced by an exception.

Another way to see it,

- a term of $A \uplus \Delta$ may raise an exception at top-level.
- a term of $A^\Delta$ may raise an exception in some evaluation context (of $A$).

Remark: $A^\Delta$ is a broader type than $A \uplus \Delta$. 
Corruption main property

The main property that corruption enjoys is
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$$(A \to B)^\Delta = A^\Delta \to B^\Delta$$
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with that we have:

$$S : \mathbb{N} \to \mathbb{N}$$
Corruption main property

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\[(A \rightarrow B)^\Delta = A^\Delta \rightarrow B^\Delta\]

with that we have:

\[S : \mathbb{N} \rightarrow \mathbb{N} \leq (\mathbb{N} \rightarrow \mathbb{N}) \cup \Delta\]
Corruption main property

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with that we have:

\[S : \mathbb{N} \rightarrow \mathbb{N} \leq (\mathbb{N} \rightarrow \mathbb{N}) \uplus \Delta \leq (\mathbb{N} \rightarrow \mathbb{N})^\Delta\]
Corruption main property

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with that we have:

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\[(A \rightarrow B)^\Delta = A^\Delta \rightarrow B^\Delta\]

with that we have:

\[S : \mathbb{N} \rightarrow \mathbb{N} \leq (\mathbb{N} \rightarrow \mathbb{N}) \cup \Delta \leq (\mathbb{N} \rightarrow \mathbb{N})^\Delta = \mathbb{N}^\Delta \rightarrow \mathbb{N}^\Delta\]

and finally:

\[S (\text{raise } \varepsilon) : \mathbb{N}\{\varepsilon}\]
Typing rules of $Fx$

\[
\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \quad (ax)
\]

\[
\frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x. M : A \rightarrow B} \quad (abs)
\]

\[
\frac{\Gamma \vdash M : A \rightarrow B \quad \Gamma \vdash N : A}{\Gamma \vdash M \, N : B} \quad (app)
\]

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\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \quad (ax)
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Typing rules of $Fx$

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\frac{\Gamma \vdash M : A \to B \quad \Gamma \vdash N : A}{\Gamma \vdash M \ N : B} \quad \text{(app)}
\]

\[
\frac{\Gamma \vdash M : A \quad \alpha \notin FV(\Gamma)}{\Gamma \vdash \forall \alpha. M : A} \quad \text{(gen)}
\]
Typing rules of $Fx$

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\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \quad (ax)
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\[
\frac{\Gamma \vdash M : A \quad \alpha \notin \text{FV}(\Gamma)}{\Gamma \vdash M : \forall \alpha. A} \quad (gen)
\]

\[
\frac{\Gamma \vdash M : A \quad A \leq B}{\Gamma \vdash M : B} \quad (subs)
\]
Typing rules of $F_x$

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\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \quad (ax)
\]

\[
\frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x. M : A \to B} \quad (abs)
\]

\[
\frac{\Gamma \vdash M : A \to B \quad \Gamma \vdash N : A}{\Gamma \vdash M \ N : B} \quad (app)
\]

\[
\frac{\Gamma \vdash M : A \quad \alpha \notin FV(\Gamma)}{\Gamma \vdash M : \forall \alpha. A} \quad (gen)
\]

\[
\frac{\Gamma \vdash M : A \quad A \leq B}{\Gamma \vdash M : B} \quad (subs)
\]

\[
\frac{\Gamma \vdash 0 : \mathbb{N}}{(zero)}
\]

\[
\frac{\Gamma \vdash S : \mathbb{N} \to \mathbb{N}}{(succ)}
\]

\[
\frac{\Gamma \vdash \text{Rec} : \forall \alpha. \alpha \to (\mathbb{N} \to \alpha \to \alpha) \to \mathbb{N} \to \alpha}{(rec)}
\]
Typing rules of $Fx$ continued

$$\Gamma \vdash \text{raise } \varepsilon : \emptyset \cup \{\varepsilon\} \quad \text{(raise)}$$

$$\emptyset \equiv \forall \alpha. \alpha$$
Typing rules of $Fx$ continued

$$\Gamma \vdash \text{raise} \varepsilon : \emptyset \uplus \{\varepsilon\} \quad (\text{raise})$$

$$\emptyset \equiv \forall \alpha. \alpha$$

$$\Gamma \vdash M : A \uplus \{\varepsilon\} \quad \Gamma \vdash N : A$$

$$\Gamma \vdash \text{try} M \text{ with } \varepsilon \mapsto N : A \quad (\text{try})$$
Typing rules of $Fx$ continued

$\Gamma \vdash \text{raise}\, \varepsilon : \emptyset \uplus \{\varepsilon\}$ (raise)

$\emptyset \equiv \forall \alpha. \alpha$

$\Gamma \vdash \text{try}\, M \text{ with } \varepsilon \mapsto N : A$ (try)

$\Gamma \vdash \text{eval} : \mathbb{N}^\Delta \rightarrow \mathbb{N} \uplus \Delta$ (eval)
First subtyping rules: $F_\eta$

\[ A \leq A \quad (st-id) \]
\[ A \leq B \quad B \leq C \quad \Rightarrow \quad A \leq C \quad (st-trans) \]
First subtyping rules: $F_\eta$

\[
\frac{}{A \leq A} \quad (st\text{-}id) \quad \frac{A \leq B \quad B \leq C}{A \leq C} \quad (st\text{-}trans)
\]

\[
\frac{A' \leq A \quad B \leq B'}{A \to B \leq A' \to B'} \quad (st\text{-}arrow)
\]
First subtyping rules: $F\eta$

\[
\frac{}{A \leq A} \quad (st-id) \quad \frac{A \leq B}{A \leq C} \quad (st-trans)
\]

\[
\frac{A' \leq A \quad B \leq B'}{A \rightarrow B \leq A' \rightarrow B'} \quad (st-arrow)
\]

\[
\frac{A \leq B \quad \alpha \notin FV(A)}{A \leq \forall \alpha. B} \quad (f-gen) \quad \frac{\forall \alpha. A \leq A\{\alpha := B\}}{(f-inst)}
\]

\[
\frac{\alpha \notin FV(A)}{\forall \alpha. (A \rightarrow B) \leq A \rightarrow \forall \alpha. B} \quad (f-arr)
\]
Subtyping rules: $A \cup \Delta$

$A \leq A \cup \Delta$ (ex-uni)
Subtyping rules: $A \cup \Delta$

$A \leq A \cup \Delta$ (ex-uni)

$A \cup \emptyset \leq A$ (ex-noex)
Subtyping rules: $A \uplus \Delta$

$$A \leq A \uplus \Delta \quad (ex\text{-}uni)$$

$$A \uplus \emptyset \leq A \quad (ex\text{-}noex)$$

$$A \leq B \quad \frac{}{A \uplus \Delta \leq B \uplus \Delta} \quad (ex\text{-}ctx)$$
Subtyping rules: $A \cup \Delta$

- **ex-uni**
  
  $A \leq A \cup \Delta$

- **ex-noex**
  
  $A \cup \emptyset \leq A$

- **ex-ctx**
  
  $A \leq B \\ A \cup \Delta \leq B \cup \Delta$

- **ex-fallu**
  
  $\forall \alpha. (A \cup \Delta) \leq (\forall \alpha. A) \cup \Delta$

- **eq-uu**
  
  $(A \cup \Delta) \cup \Delta' = A \cup (\Delta \cup \Delta')$
Subtyping rules: $A^\Delta$

\[
A \uplus \Delta \leq A^\Delta \quad (\text{ex-corrupt})
\]
Subtyping rules: $A^\Delta$

\[
A \cup \Delta \leq A^\Delta \quad (ex\text{-}corrupt)
\]

\[
\emptyset^\Delta \leq \emptyset \cup \Delta \quad (ex\text{-}empty)
\]
Subtyping rules: $A^\Delta$

\[
\begin{align*}
A \cup \Delta & \leq A^\Delta \quad \text{(ex-corrupt)} \\
\emptyset^\Delta & \leq \emptyset \cup \Delta \quad \text{(ex-empty)} \\
\forall \alpha. A^\Delta & \leq (\forall \alpha. A)^\Delta \quad \text{(ex-fallc)}
\end{align*}
\]
Subtyping rules: $A^\Delta$

- $A \cup \Delta \leq A^\Delta$ (ex-corrupt)
- $\emptyset^\Delta \leq \emptyset \cup \Delta$ (ex-empty)
- $\forall \alpha. A^\Delta \leq (\forall \alpha. A)^\Delta$ (ex-fallc)

- $(A^\Delta)^\Delta' = A^{(\Delta \cup \Delta')}$ (eq-cc)
- $(A \cup \Delta)^\Delta' = A^\Delta' \cup \Delta$ (eq-uc)
Subtyping rules: $A^\Delta$

\[
\frac{}{A \cup \Delta \leq A^\Delta} \quad (ex\text{-}corrupt) \quad \frac{}{\emptyset^\Delta \leq \emptyset \cup \Delta} \quad (ex\text{-}empty)
\]

\[
\frac{}{\forall \alpha. A^\Delta \leq (\forall \alpha. A)^\Delta} \quad (ex\text{-}fallc)
\]

\[
\frac{}{(A^\Delta)^\Delta' = A^{(\Delta \cup \Delta')}} \quad (eq\text{-}cc) \quad \frac{}{(A \cup \Delta)^\Delta' = A^{\Delta'} \cup \Delta} \quad (eq\text{-}uc)
\]

\[
\frac{}{(A \to B)^\Delta = A^\Delta \to B^\Delta} \quad (eq\text{-}arrc)
\]
Examples

\[ \text{pred} \equiv \text{Rec} (\text{raise } \varepsilon) (\lambda x. \lambda y. x) : \]
Examples

\[
pred \equiv \text{Rec}(\text{raise } \varepsilon)(\lambda x. \lambda y. x) : \mathbb{N} \rightarrow \mathbb{N} \uplus \{\varepsilon\}
\]
Examples

\[
pred \equiv \text{Rec} (\text{raise } \varepsilon) (\lambda x. \lambda y. x) : \mathbb{N} \rightarrow \mathbb{N} \cup \{\varepsilon\}
\]
\[
pred' \equiv \lambda n. \text{try} (\text{pred } n) \text{ with } \varepsilon \mapsto 0 : \mathbb{N} \rightarrow \mathbb{N}
\]
Examples

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pred \equiv \text{Rec} (\text{raise } \varepsilon) (\lambda x. \lambda y. x) : \mathbb{N} \to \mathbb{N} \cup \{\varepsilon\}
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pred' \equiv \lambda n. \text{try} (\text{pred } n) \text{ with } \varepsilon \mapsto 0 : \mathbb{N} \to \mathbb{N}
\]
\[
\text{add2} \equiv \lambda n. S (S n)
\]
Examples

\[\text{pred} \equiv \text{Rec}(\text{raise } \varepsilon)(\lambda x. \lambda y. x) : \mathbb{N} \rightarrow \mathbb{N} \uplus \{\varepsilon\}\]
\[\text{pred}' \equiv \lambda n. \text{try}(\text{pred } n) \text{ with } \varepsilon \mapsto 0 : \mathbb{N} \rightarrow \mathbb{N}\]
\[\text{add2} \equiv \lambda n. S(Sn)\]

We can define second order list and then:
Examples

\[
pred \equiv \text{Rec}\ (\text{raise}\ \varepsilon)\ (\lambda x.\ \lambda y.\ x) : \ \mathbb{N} \to \mathbb{N} \cup \{\varepsilon\}
\]

\[
pred' \equiv \lambda n.\ \text{try}\ (\text{pred}\ n)\ \text{with}\ \varepsilon \mapsto 0 : \ \mathbb{N} \to \mathbb{N}
\]

\[
\text{add2} \equiv \lambda n.\ S\ (S\ n)
\]

We can define second order list and then:

\[
\text{map}\ (\lambda n.\ \text{add2}\ (\text{pred}\ n))\ [3;\ 0;\ 4]
\]
Examples

\[
\begin{align*}
\text{pred} & \equiv \text{Rec } (\text{raise } \varepsilon) (\lambda x. \lambda y. x) : \mathbb{N} \to \mathbb{N} \cup \{ \varepsilon \} \\
\text{pred}' & \equiv \lambda n. \text{try } (\text{pred } n) \text{ with } \varepsilon \mapsto 0 : \mathbb{N} \to \mathbb{N} \\
\text{add2} & \equiv \lambda n. \text{S } (\text{S } n)
\end{align*}
\]

We can define second order list and then:

\[
\text{map } (\lambda n. \text{add2 } (\text{pred } n)) [3; 0; 4] > [4; \text{S } (\text{S } (\text{raise } \varepsilon )); 5]
\]
Examples

\[
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\[
\text{head} [4; S (S (\text{raise } \varepsilon)); 5] > 4
\]
1 Introduction

2 Presenting $F_x$
   - The terms side
   - The types side

3 A realisability model of $F_x$

4 Conclusion and perspectives
A realisability model

We have developed a realisability model of $Fx$. 
A realisability model

We have developed a realisability model of $F_x$. It is designed using orthogonality techniques.
We have developed a realisability model of $Fx$.

It is designed using orthogonality techniques.

The interpretations of $A ⋆ \Delta$ and $A^\Delta$ follow the idea that
- $A ⋆ \Delta$ may raise exceptions at top-level.
- $A^\Delta$ may raise exceptions in some evaluation context.
Arrow types interpretation is not standard

With corruption, the arrow types is not the usual realisability one anymore.
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With corruption, the arrow types is not the usual realisability one anymore.

If we denote by $\overset{r}{\to}$ the realisability arrow, our arrow is understand as:

$$A \rightarrow B = \bigcap_{\Delta \subseteq \mathcal{E}} (A^{\Delta} \overset{r}{\to} B^{\Delta})$$
Theorem (Model soundness)

The model is sound.
Model properties

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Corollary (Type safety)

If $M$ is a term such that $\vdash M : \mathbb{N}$, then $M \triangleright^* S^n 0$. 
Model properties

**Theorem (Model soundness)**

The model is sound.

**Corollary (Type safety)**

If $M$ is a term such that $\vdash M : \mathbb{N}$, then $M \leadsto^* S^n 0$.

**Corollary (Weak normalization)**

Terms of $Fx$ are weakly normalizing.
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A mechanism of exceptions adapted to call-by-name evaluation and a way to precisely type it based on subtyping.
Conclusion

- A mechanism of exceptions adapted to call-by-name evaluation and a way to precisely type it based on subtyping.

- It should be adaptable to dependent types.
A mechanism of exceptions adapted to call-by-name evaluation and a way to precisely type it based on subtyping.

It should be adaptable to dependent types.

That’s all, thanks for your attention.