Randomization Techniques for Secure Computation and Parallel Cryptography

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Course based on joint works with

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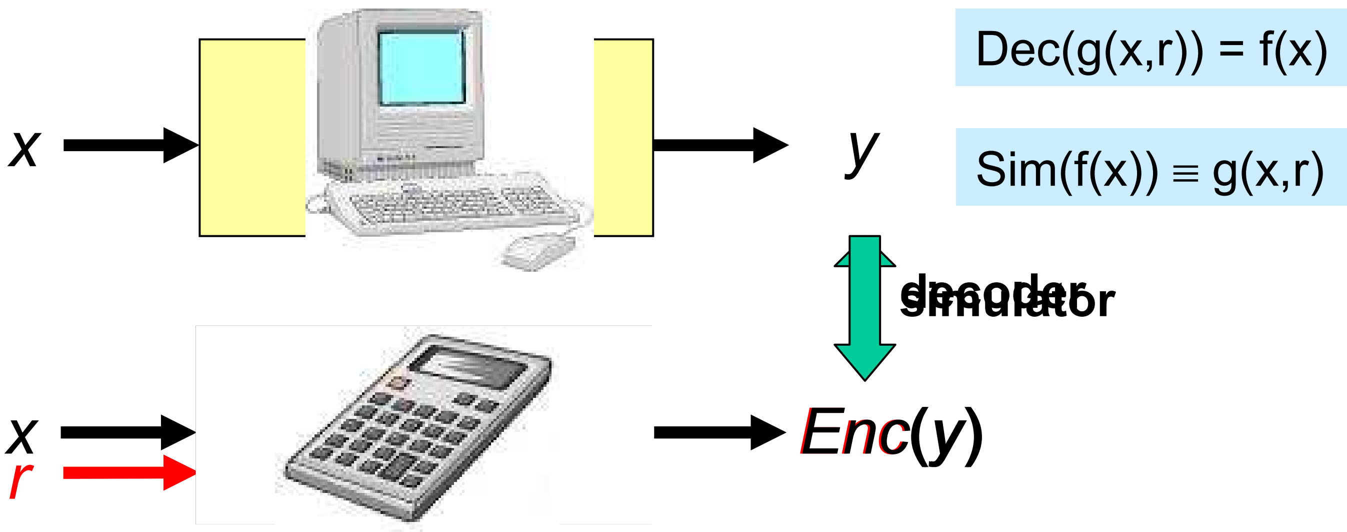
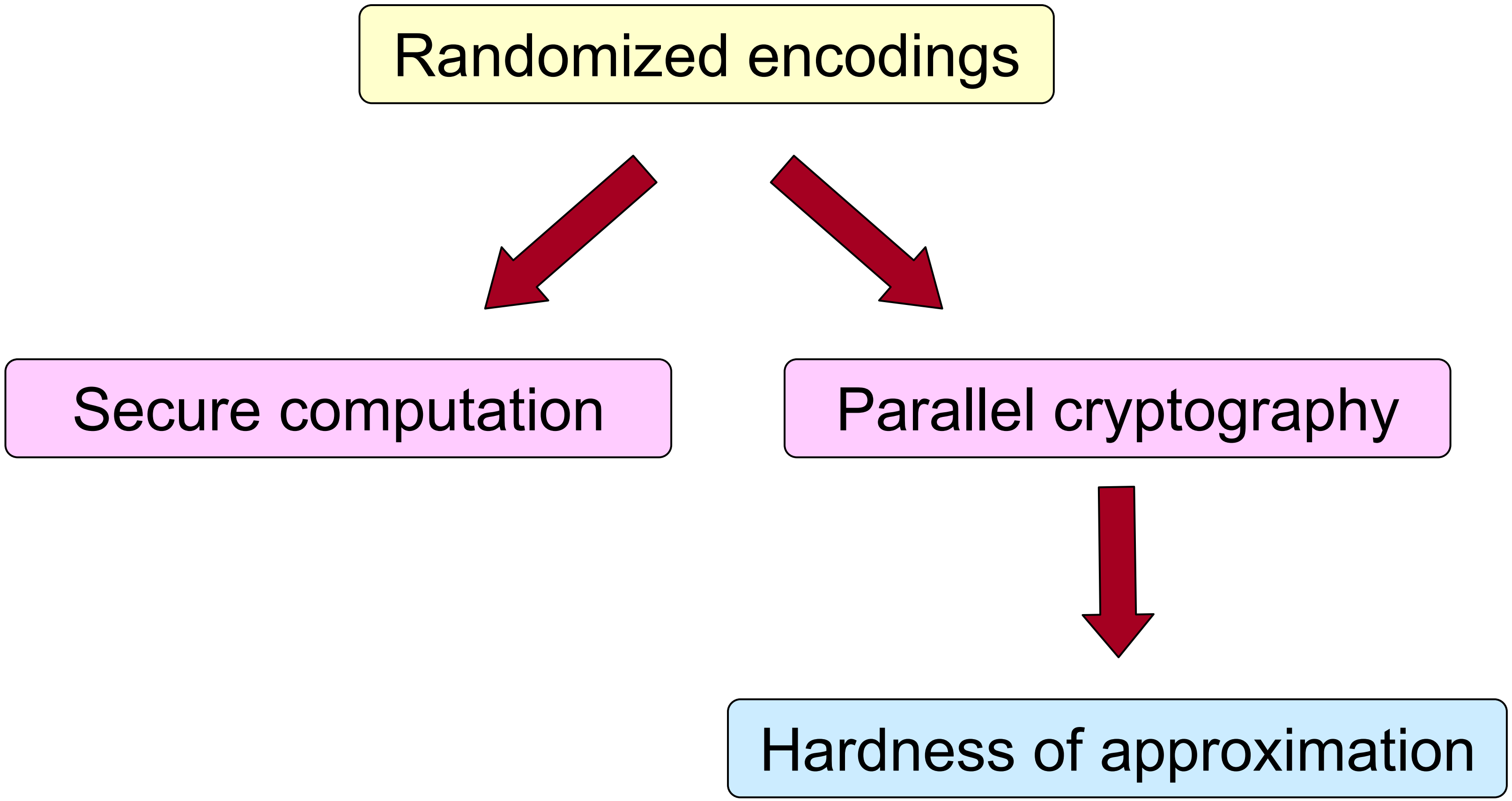
(…….,FOCS 00, ICALP 02, FOCS 04, CCC 05, RANDOM 06, CRYPTO 07)

The Basic Question

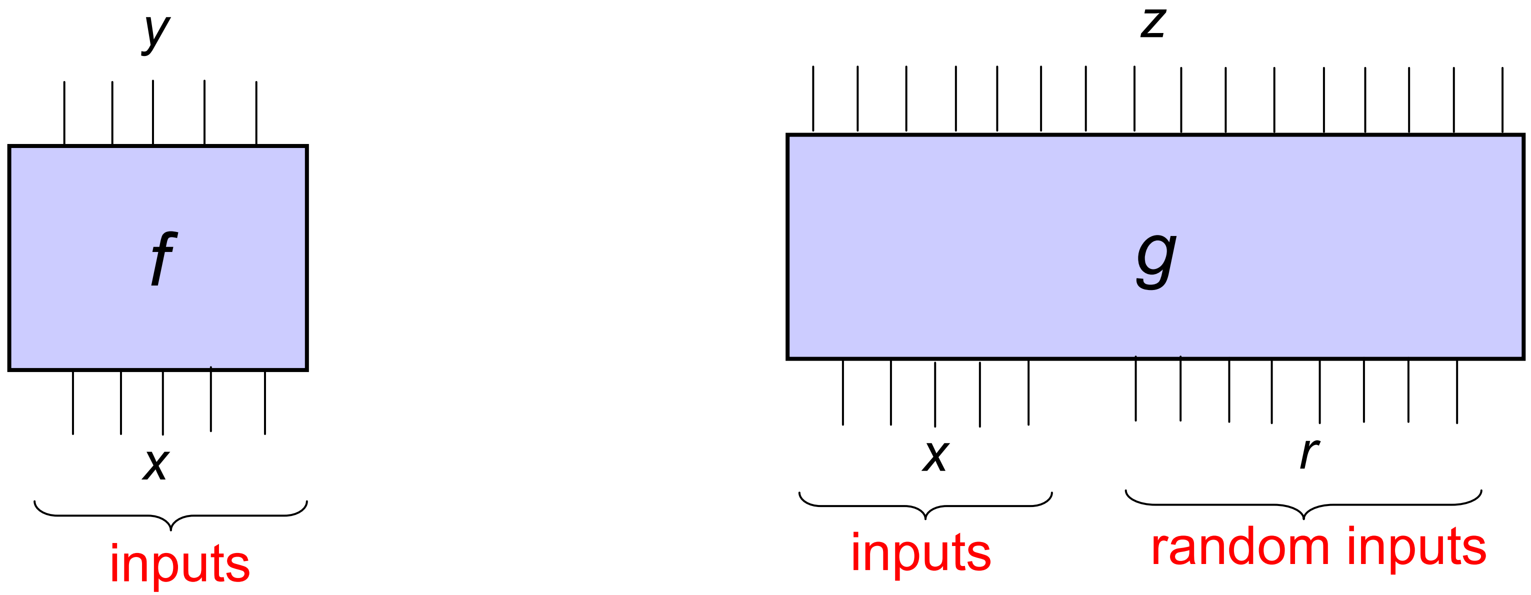


- *g* is a "randomized encoding" of *f*
  - Nontrivial relaxation of computing *f*
- Hope:
  - *g* can be "simpler" than *f*
    (meaning of "simpler" determined by application)
  - *g* can be used as a substitute for *f*
  - *g* inherits security properties of *f*

Applications at a Glance



Randomized Encoding - Syntax

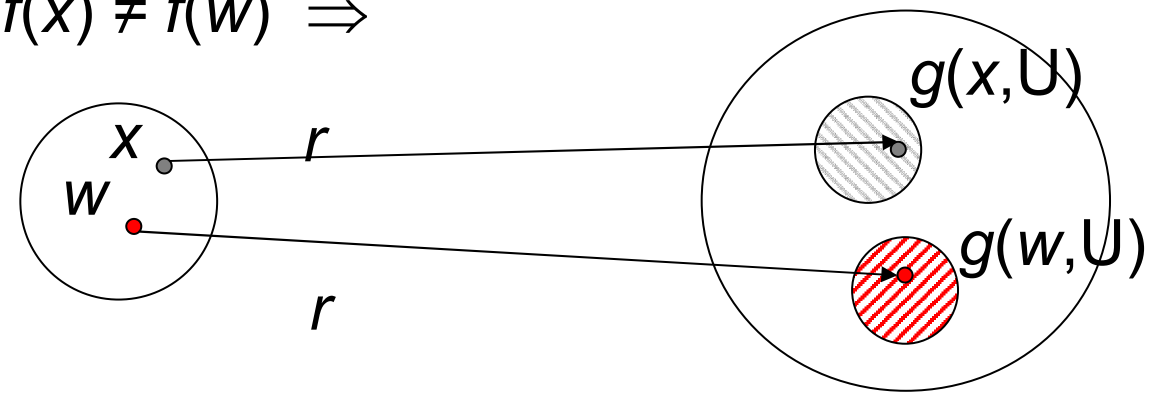


$f(x)$ is encoded by $g(x,r)$

Randomized Encoding - Semantics

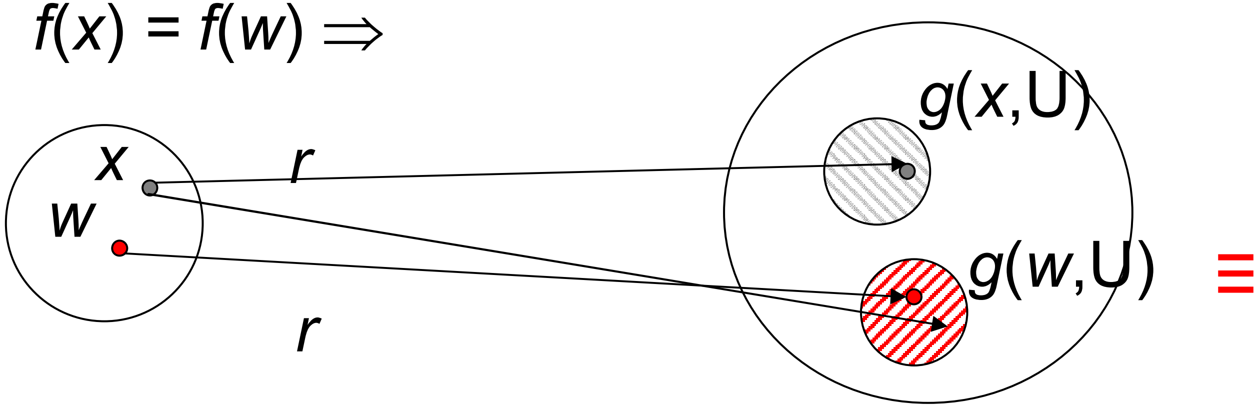
- Correctness: $f(x)$ can be efficiently decoded from $g(x,r)$.

  $f(x) \neq f(w) \Rightarrow$



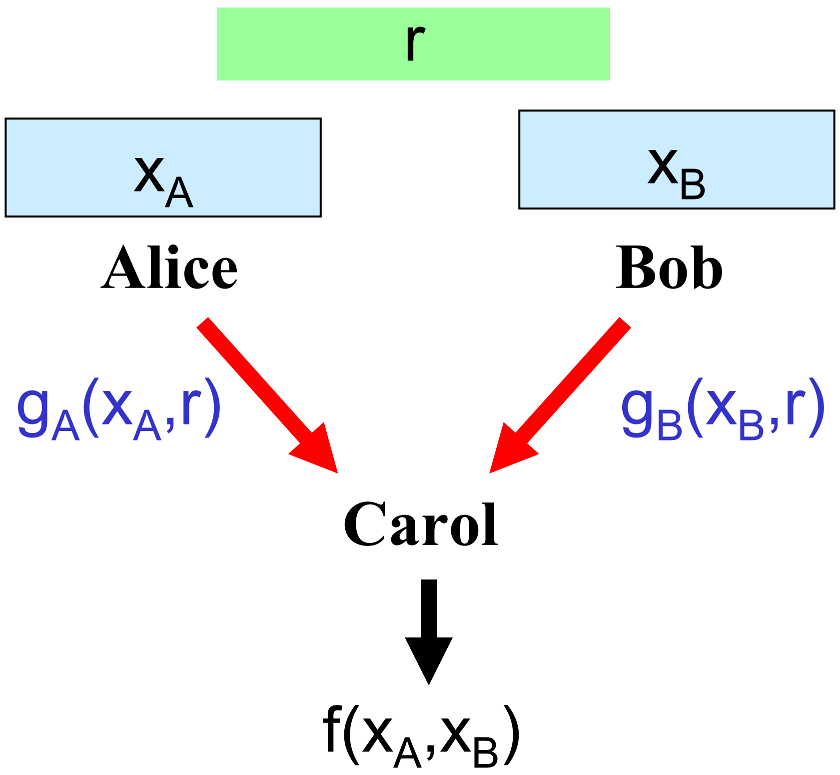
- Privacy: $\exists$ efficient simulator Sim such that Sim($f(x)$) $\equiv g(x,\mathrm{U})$
  - $g(x,\mathrm{U})$ depends only on $f(x)$

  $f(x) = f(w) \Rightarrow$



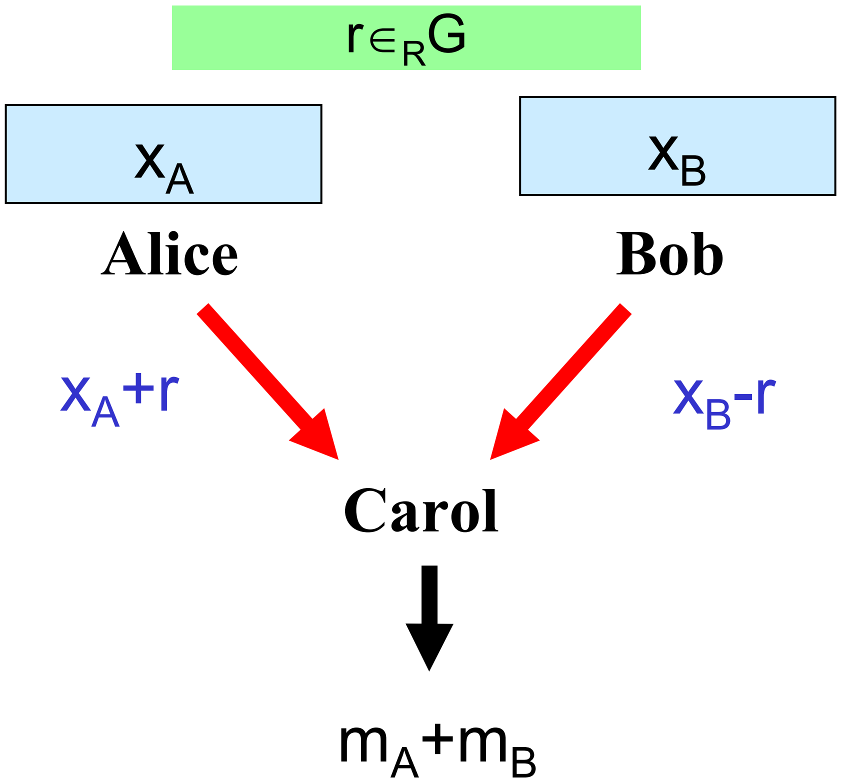
Notions of Simplicity - I

- Application: "minimal model for secure computation" [Feige-Kilian-Naor 94, …]
- 2-decomposability: $g((x_A,x_B),r)=(g_A(x_A,r),g_B(x_B,r))$



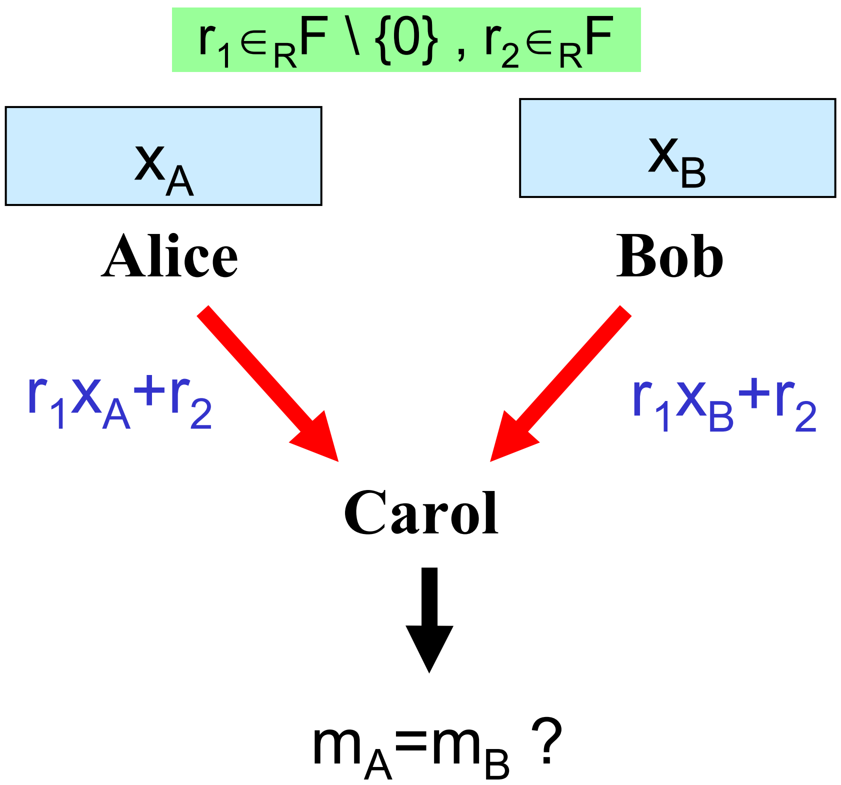
Example: sum

- $f(x_A,x_B) = x_A+x_B$ ($x_A,x_B \in$ finite group G)



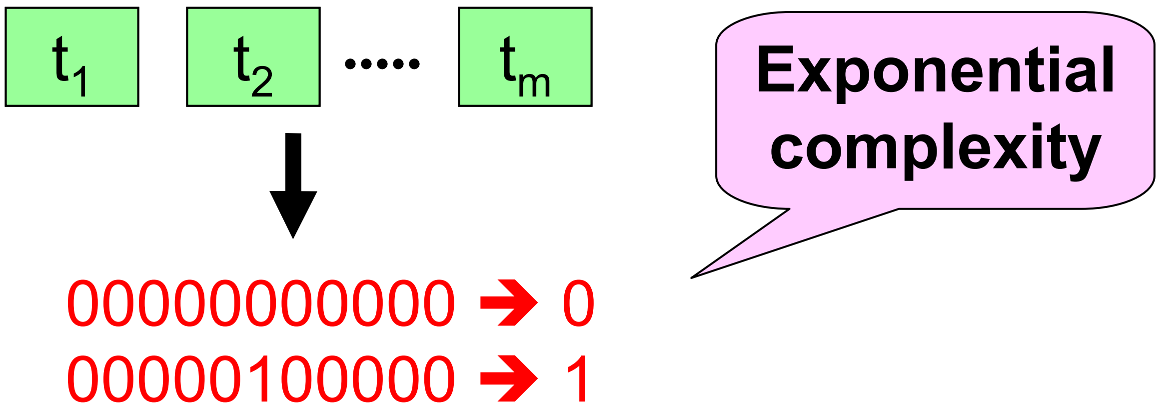
Example: equality

- $f(x_A,x_B)$ = equality ($x_A,x_B \in$ finite field F)



Example: ANY function

- $f(x_A,x_B) = x_A \wedge x_B$ $(x_A,x_B \in \{0,1\})$
  - Reduction to equality: $x_A \rightarrow 0/1$, $x_B \rightarrow 2/1$
- General boolean f: write as **disjoint** 2-DNF
  - $f(x_A,x_B) = \vee_{(a,b):f(a,b)=1} (x_A=a \wedge x_B=b) = t_1 \vee t_2 \vee \ldots \vee t_m$

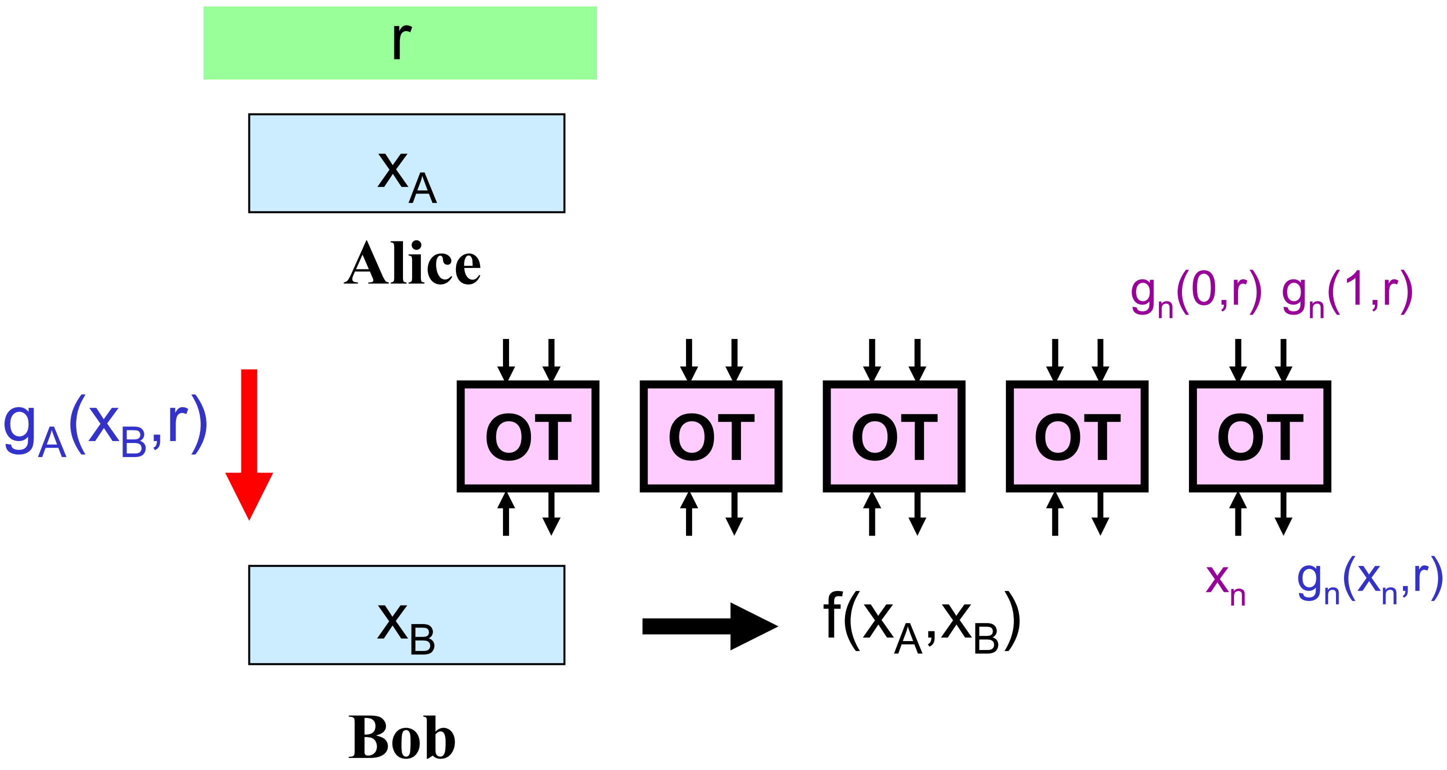


Notions of Simplicity - II

- Full decomposability:

  $g((x_1,\ldots,x_n),r)=(g_1(x_1,r),\ldots,g_n(x_n,r))$

  – Application: Basing SFE on OT [Kilian 88, ...]



Example: iterated group product

- Abelian case
  - $f(x_1,\ldots,x_n)=x_1+x_2+\ldots+x_n$
  - $g(x, (r_1,\ldots,r_{n-1})) =$

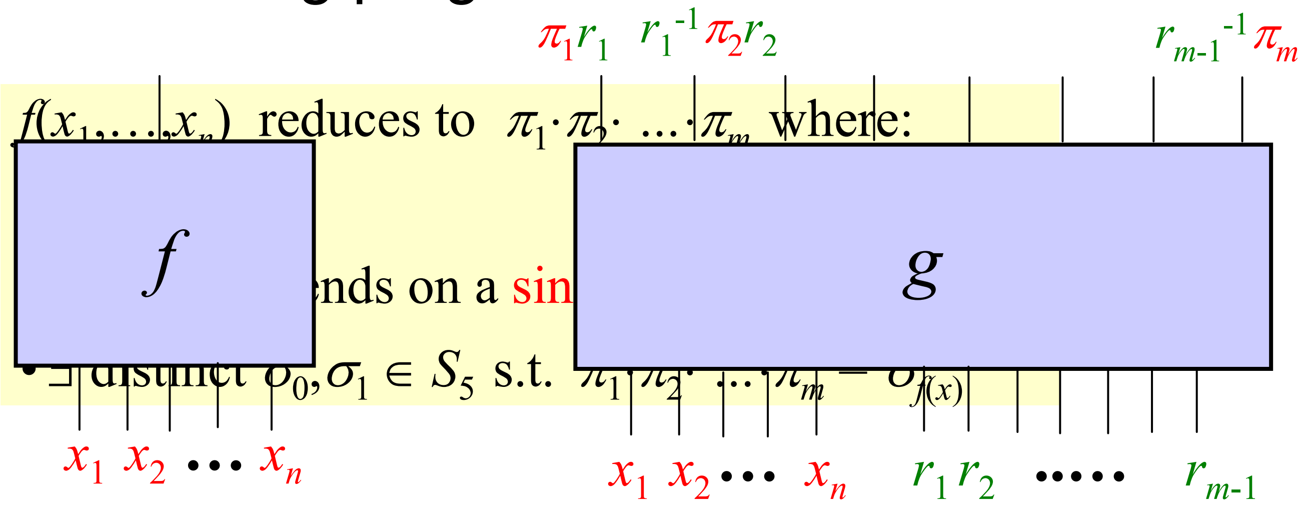
x_1+r_1 | x_2+r_2 | $\ldots$ | $x_{n-1}+r_{n-1}$ | $x_n-r_1-\ldots-r_{n-1}$

- General case [Kilian 88]
  - $f(x_1,\ldots,x_n)=x_1x_2\cdots x_n$
  - $g(x, (r_1,\ldots,r_{n-1})) =$

x_1r_1 | $r_1^{-1}x_2r_2$ | $r_2^{-1}x_2r_3$ | $\ldots$ | $r_{n-2}^{-1}x_{n-1}r_{n-1}$ | $r_{n-1}^{-1}x_n$

Example: iterated group product

Thm [Barrington 86]
Every boolean $f \in NC^1$ can be computed by a poly-length, width-5 branching program.



Encoding iterated group product

- Every output bit of g depends on just a single bit of x

$\pi_1\cdot\pi_2\cdot\pi_3\cdot\ldots\cdot\pi_m \Rightarrow \pi_1 r_1 \quad r_1^{-1}\pi_2 r_2 \quad r_2^{-1}\pi_3 r_3 \quad \ldots \quad r_{m-1}^{-1}\pi_m$

   ➢ Efficient fully decomposable encoding for every $f \in NC^1$

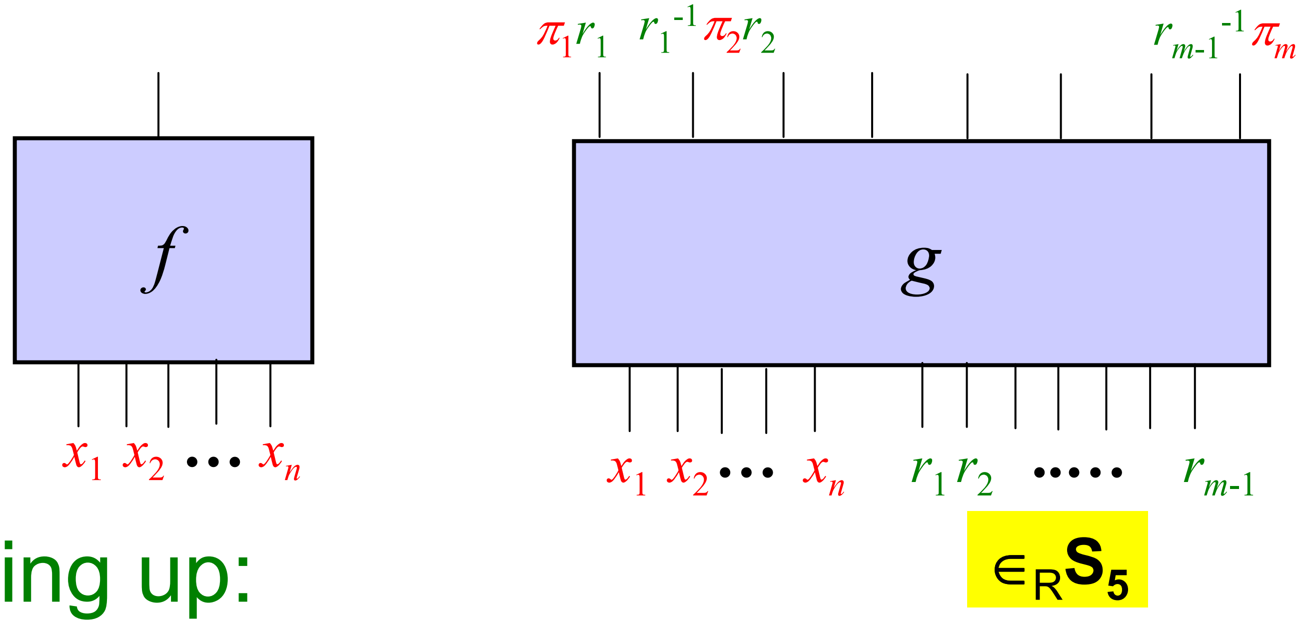
Notions of Simplicity - III

- Low degree: $g(x,r)$ = vector of degree-d poly in x,r over F
  - aka "Randomizing Polynomials" [IK00,…]
  - Application: round-efficient MPC

- Motivating observation:
  Low-degree functions are easy to distribute!
  - Round complexity of MPC protocols [BGW88,CCD88,CDM00,…]
    - Semi-honest model
      - $t<n/d$ ➔ 2 rounds
      - $t<n/2$ ➔ multiplicative depth + 1 = $\lceil \log d \rceil$+1 rounds
    - Malicious model
      - Optimal t ➔ $O(\log d)$ rounds

Examples

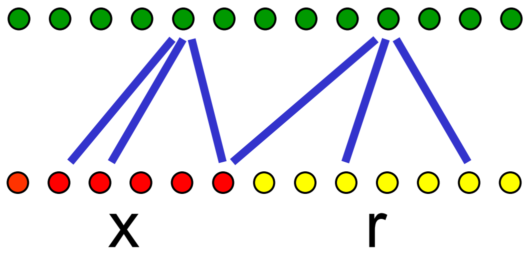
- What's wrong with previous examples?
  - Great degree in x (deg_x=1), bad degree in r



- Coming up:
  - Degree-3 encoding for every f
  - Efficient in size of branching program

Notions of Simplicity - IV

- Small locality:



  – Application: parallel cryptography!

[AIK04,AIK05,AIK07…]

- Coming up: encodings with locality 4
  – degree 3, fully decomposable
  – efficient in size of branching program

Parallel Cryptography

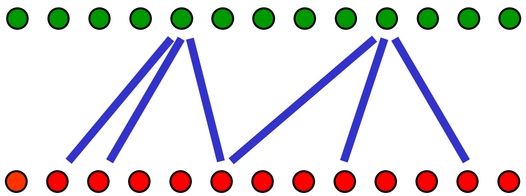
How low can we get?

poly-time
NC
log-space
NC^1
AC^0
NC^0

"Simplicity" of Cryptographic Primitives

- Can cryptographic primitives be computed by **very simple** functions?

NC^0_3

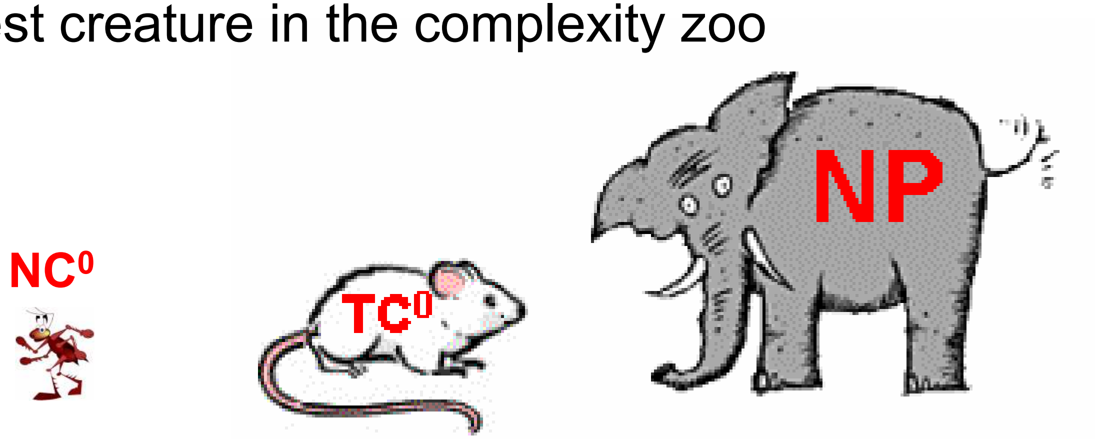


Simple = each output bit depends on $O(1)$ input bits

= const. depth circuits with bounded fan-in

= **NC^0**

- Currently the smallest creature in the complexity zoo



Cryptography in NC^0?

- Longstanding open question

  Håstad 87
  Impagliazzo Naor 89
  Goldreich 00
  Cryan Miltersen 01
  Krause Lucks 01
  Mossel Shpilka Trevisan 03

- Real-life motivation: super-fast cryptographic hardware

- Tempting conjecture:

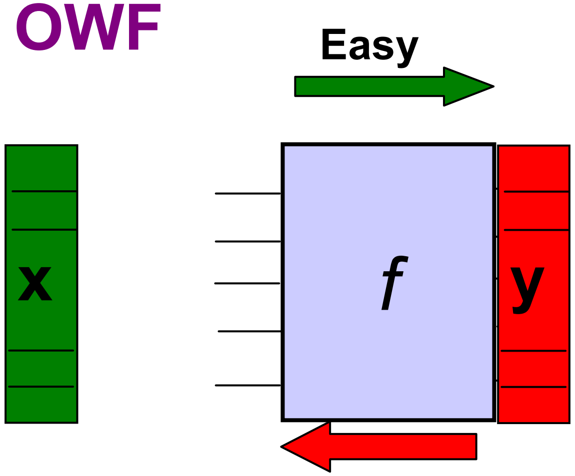
crypto hardness $\Rightarrow$ "complex" function

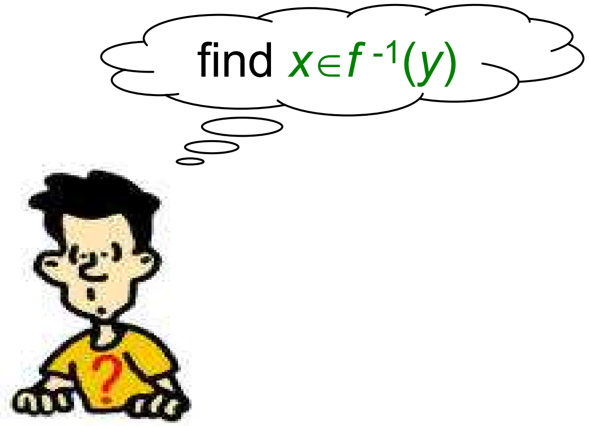
[CM]: Yes

[G]: No

Basic Primitives: One-way Function (OWF)

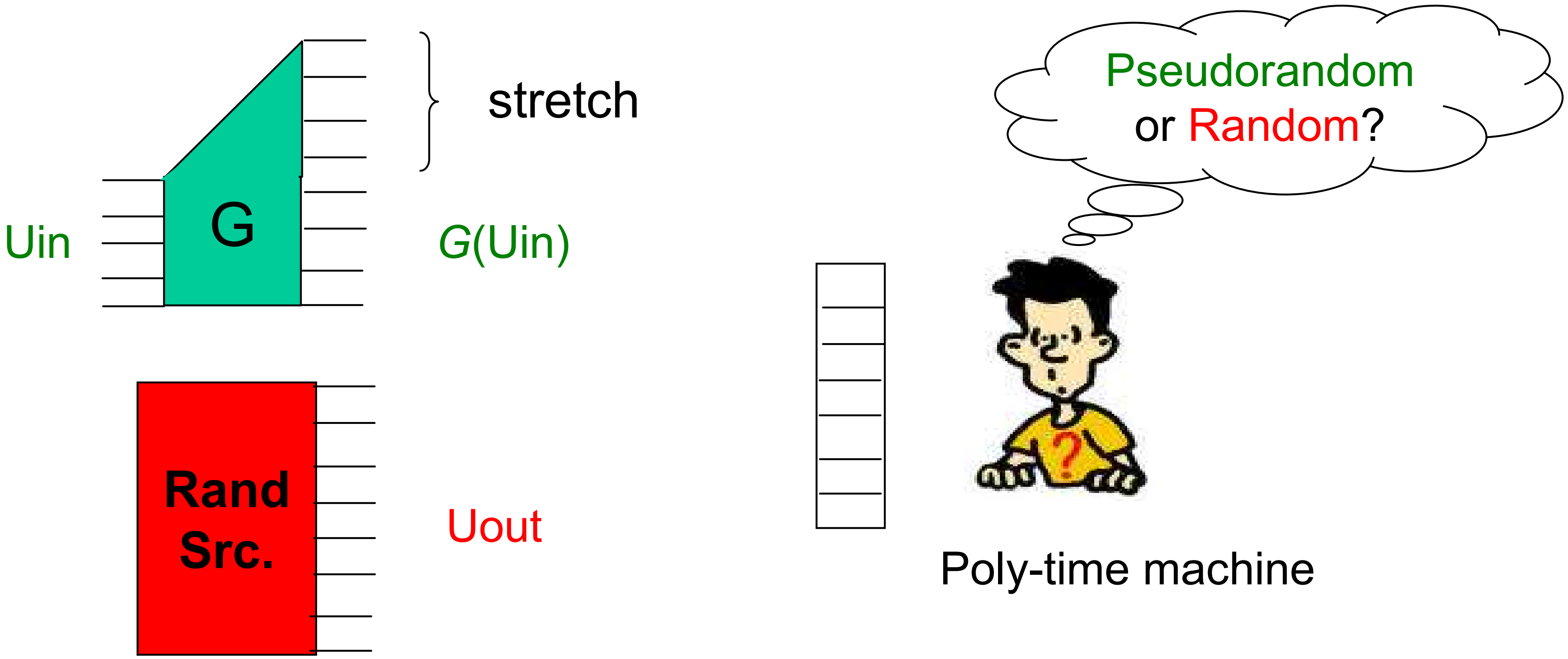
OWF





Poly-time machine

Basic Primitives: Pseudorandom Generator (PRG)



Def. PRG is minimal if stretch=1

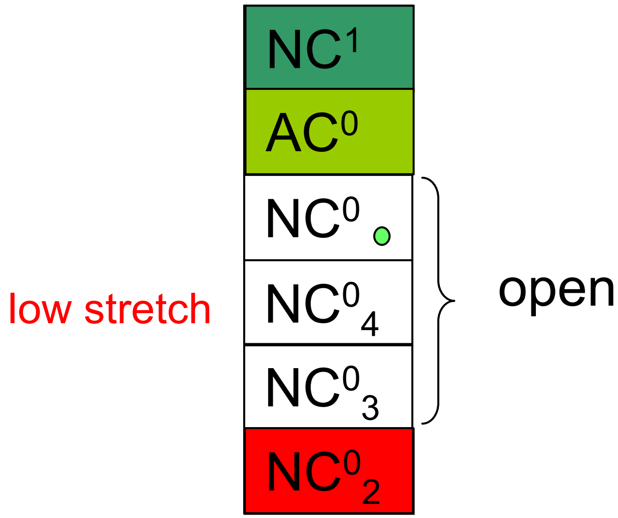
Previous Work

- Positive results
  - PRG in NC^1 from factoring, discrete-log, lattices…
  - PRF in NC^1 from factoring [Naor Reingold 97]
  - PRG (sub-lin stretch) in AC^0 from subset sum [Impagliazzo Naor 89]
- Negative results
  - No OWF in NC^0_2 [Goldreich 00, CryanMiltersen 01]
  - No PRG with large stretch in NC^0_3, NC^0_4 [CryanMiltersen01, MosselShpilkaTrevisan03]
- Permutation in NC^0 which is P-complete to invert [Håstad 87]
- Function in NC^0 which is NP-complete to invert [Agrawal Allender Rudich98]
- Heuristic construction of OWF/PRG in NC^0 [Goldreich 00 MST 03]

factoring, discrete-log, lattices, …

subset sum

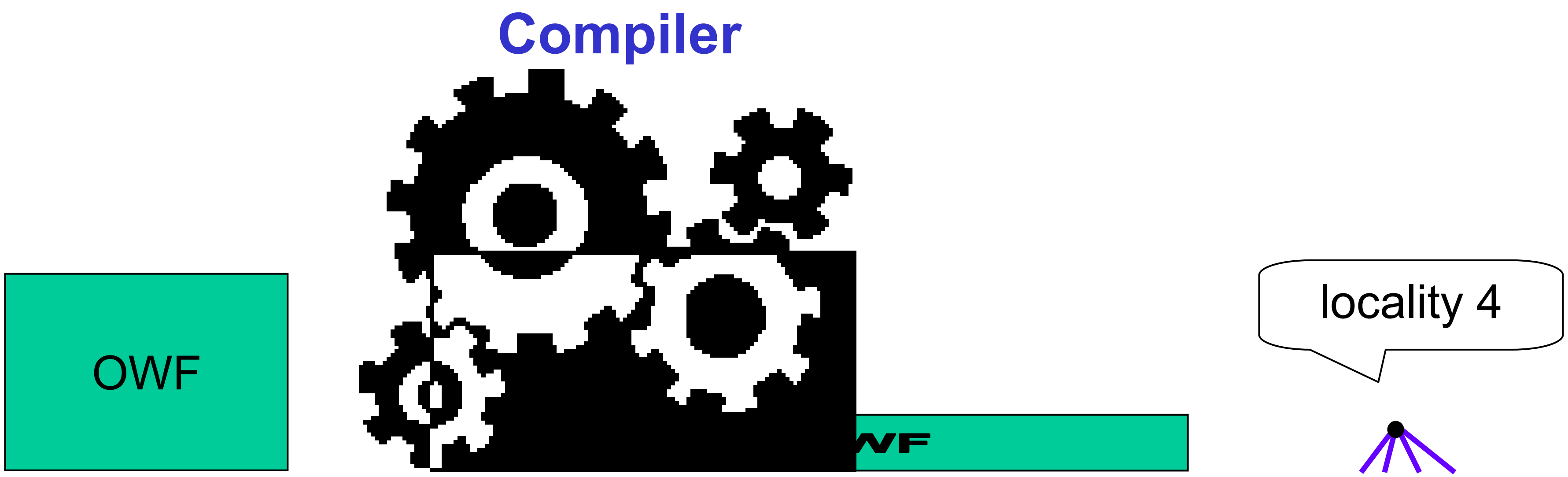
impossible



PRG / OWF

Our Approach

Compile primitives in a “relatively high” complexity class into ones in NC^0.



Sufficient Assumptions for Crypto in NC^0

Caveats:

- We g… with sub-linear st…
- decry… verification not in
  - In … ssible to decrypt/ver…
  - … commit in NC^0 with … in $NC^0[AND]$

Assuming min-PRG in NC^1

PRG
Hash

Sym-Enc
PK-Enc
Signature
Commit
NIZK

exist NC^1 $\Longrightarrow$ $\in NC^0$ [AIK04] [AIK05]

OWF
PRG
Hash

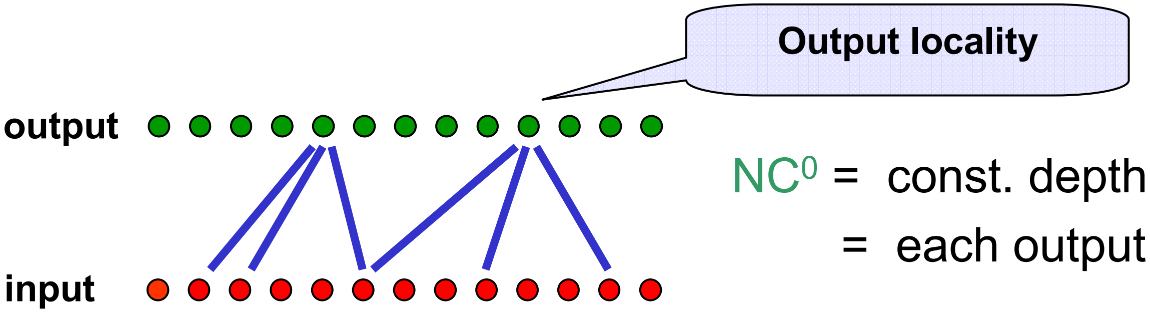
Sym-Enc
PK-Enc
Signature
Commit
NIZK

factoring, discrete-log/DDH, lattices, …

	OWF	PRG	Hash	Sym-Enc	PK-Enc	NI-Com	Sign	NIZK
P								factoring
NC^1								factoring
NC^0_4								factoring

Cryptography with Constant Input Locality

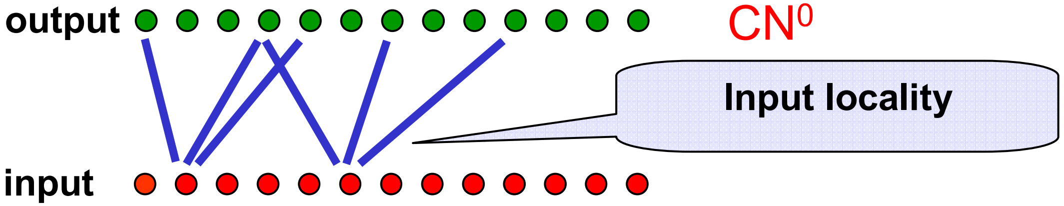
Till now we considered only NC^0 functions…



NC^0 = const. depth circuits with bounded fan-in

= each output bit depends on $O(1)$ input bits

Q: Can cryptographic primitives be realized by functions in which each input bit affects a constant number of output bits?



CN^0

Outline

1. (Long) Introduction
2. Randomized Polynomials (w/applications to round-efficient MPC)
3. Randomized Encodings (w/applications to NC^0 Cryptography)
4. Constant Input Locality
5. Computational Randomized Encodings (w/applications)
6. NC^0 Linear Stretch PRG (w/applications)