AARHUS UNIVERSITY

Secure Computation
EWSCS, Lecture 3

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What we learned this morning

- If you can **securely compute random multiplications**, you can compute any function.

- This lecture: **Oblivious Transfer** (*or how to compute random multiplications modulo 2)*
  - Definition and simple reductions
  - OT Extension
  - OT from PKE+
  - NaorPinkas OT
  - PVW OT

What we learned this morning

How to compute [z]=[xy] ?

Alice, Bob should compute

$$z_A + z_B = (x_A+x_B)(y_A+y_B)$$

$$= x_Ay_A + x_By_A + x_Ay_B + x_By_B$$

How do we compute this?

Alice can compute this

Bob can compute this

On the use of computational assumptions

- How much can we ask users to trust crypto?
  1. **Necessary** (one way functions are needed for symmetric crypto, public key crypto is probably needed for 2PC)
  2. **We must believe that some problems are hard** (e.g., breaking RSA or breaking AES). But we should not ask for more trust than needed!
  3. Construct complex systems based on well studied assumptions. Then prove (via reduction), that ***any adv that can break property X of system S can be used to solve computational problem P.***
  4. If **we believe problem P to be hard, then we conclude that system S has property X.**

The Crypto Toolbox

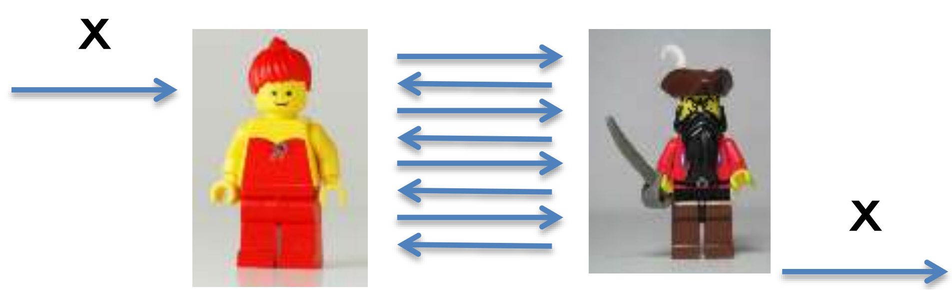
Weaker assumption ← Stronger assumption

OTP >> SKE >> PKE >> FHE >> Obfuscation

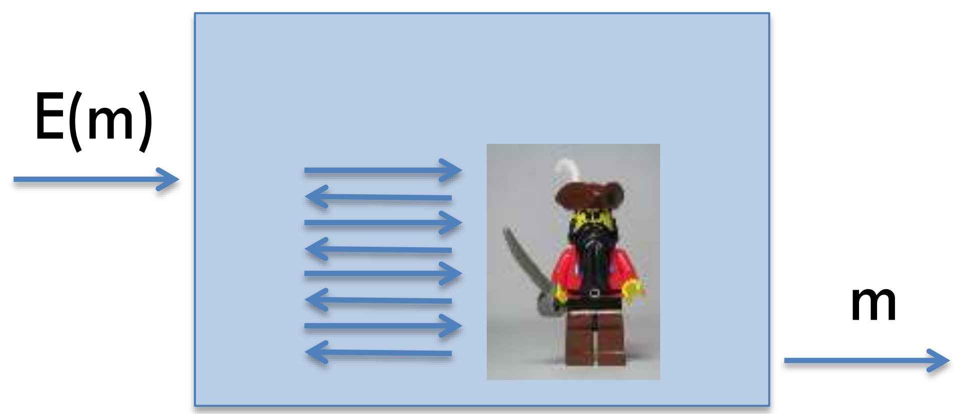
More efficient ← Less efficient

REDUCTION PROOF

- **If:** an adversary can break the security (e.g., learn the secret input x)

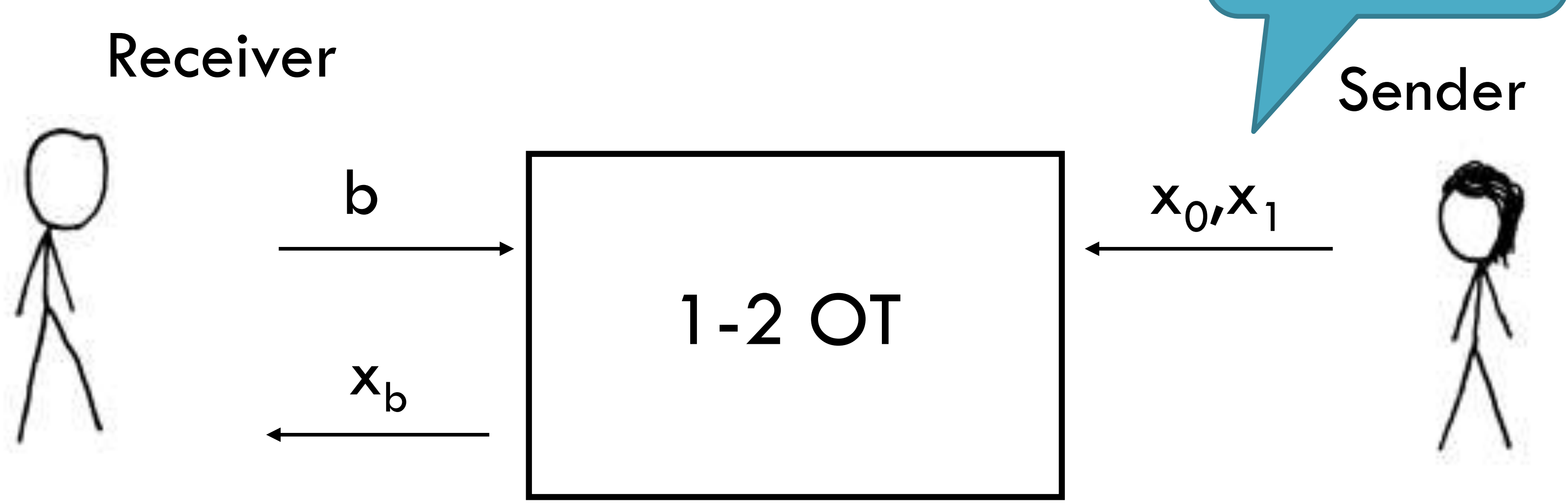


- **Then:** use this adversary as a subroutine to break the security of RSA
- **But:** we believe RSA is hard



- **So:** the protocol must be secure

Oblivious Transfer



- **Receiver** learns just one message
- **Sender** does not learn which one
- OT is a fundamental cryptographic primitive
  - **Necessary and sufficient for MPC**
  - Can be also instantiated using non crypto assumptions (noise, quantum)

The Crypto Toolbox

OT ≈ PKE

Weaker assumption

Stronger assumption

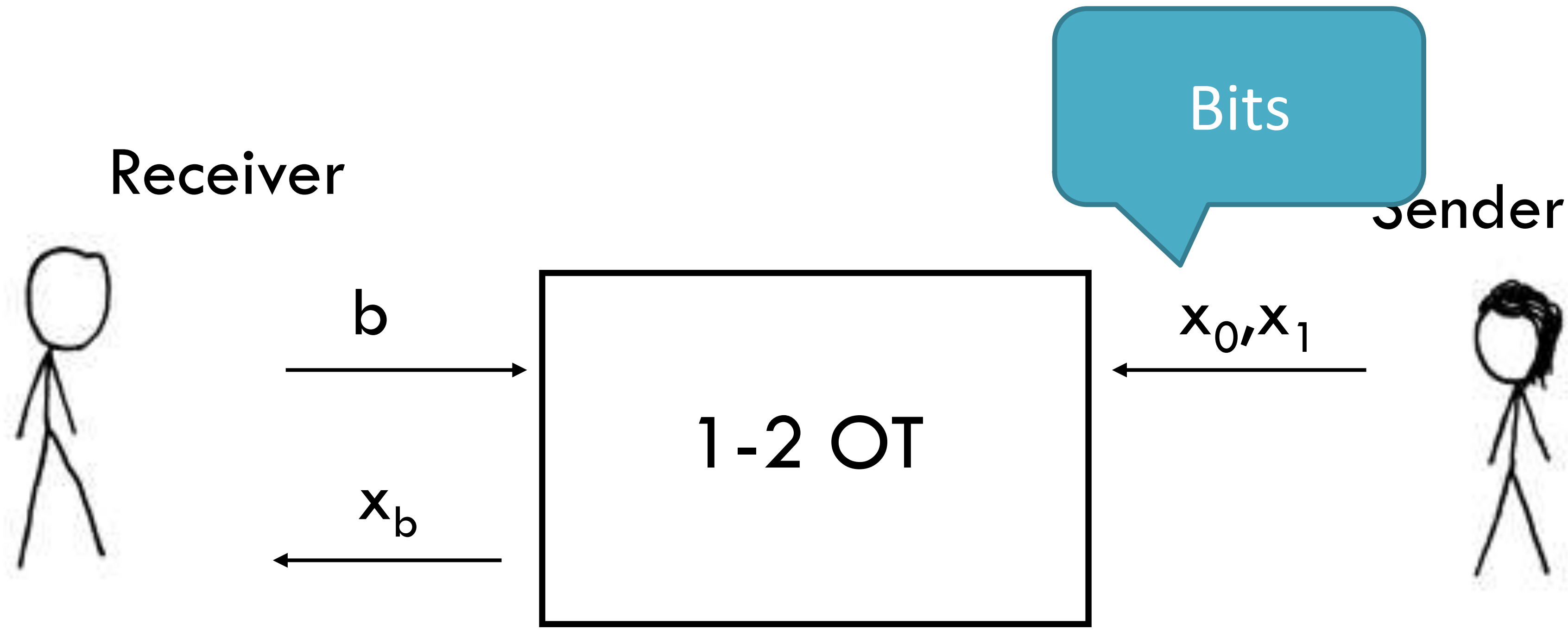
OTP >> SKE >> PKE >> FHE >> Obfuscation

More efficient

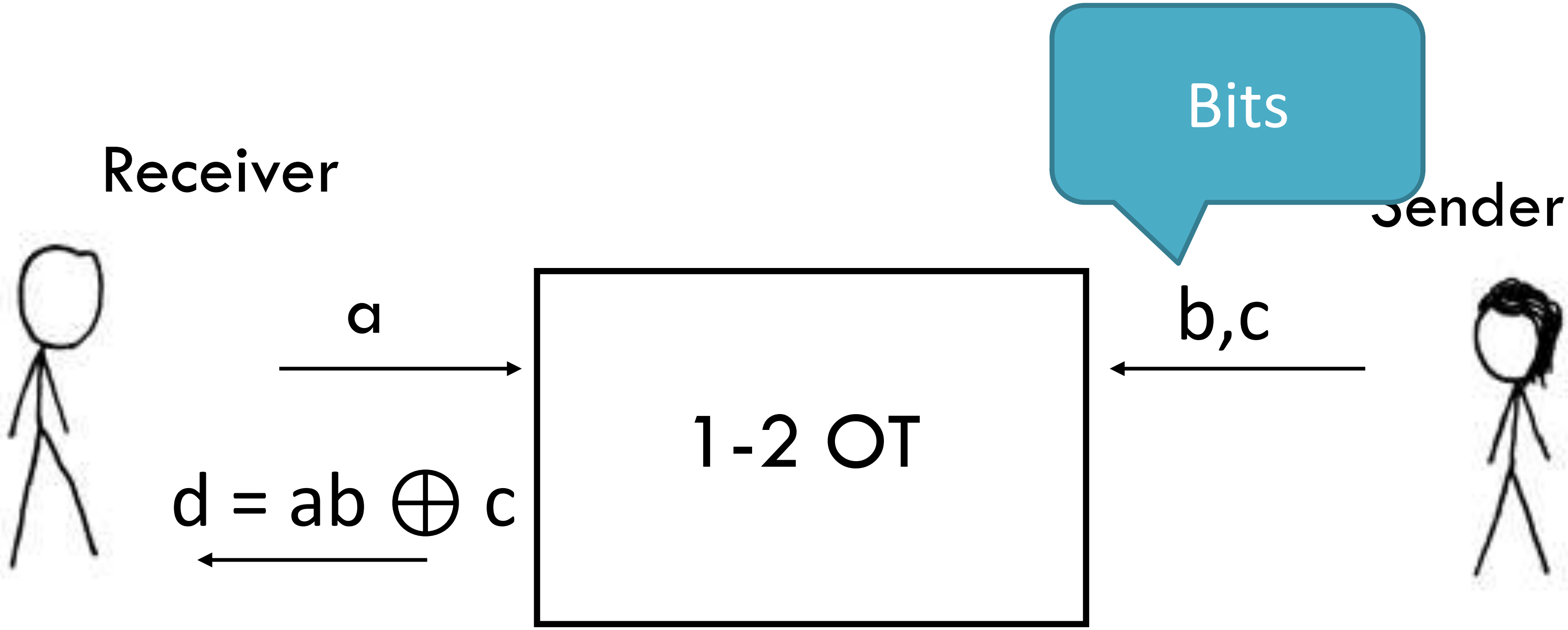
Less efficient

Simple properties of Oblivious Transfer

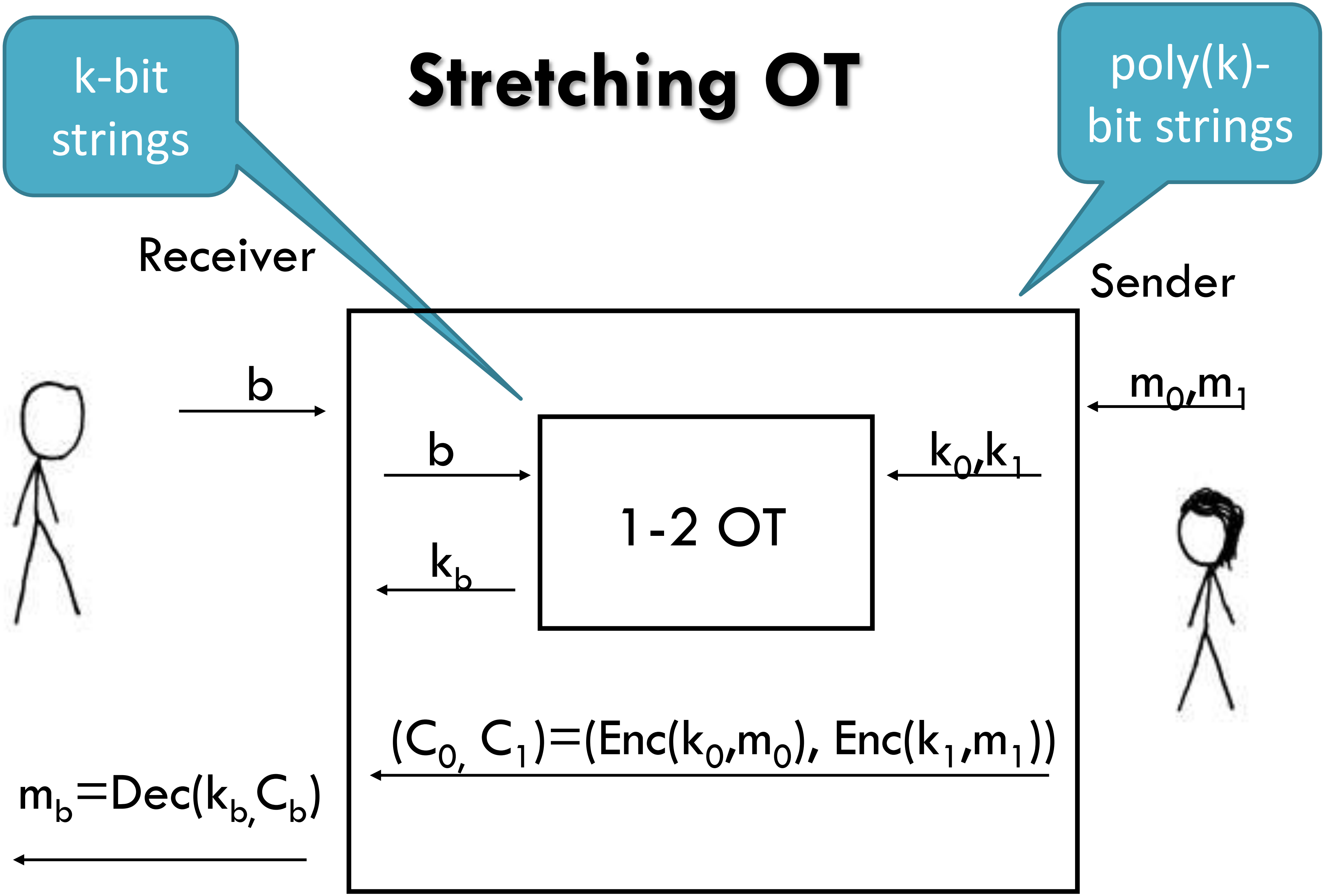
OT = shared AND



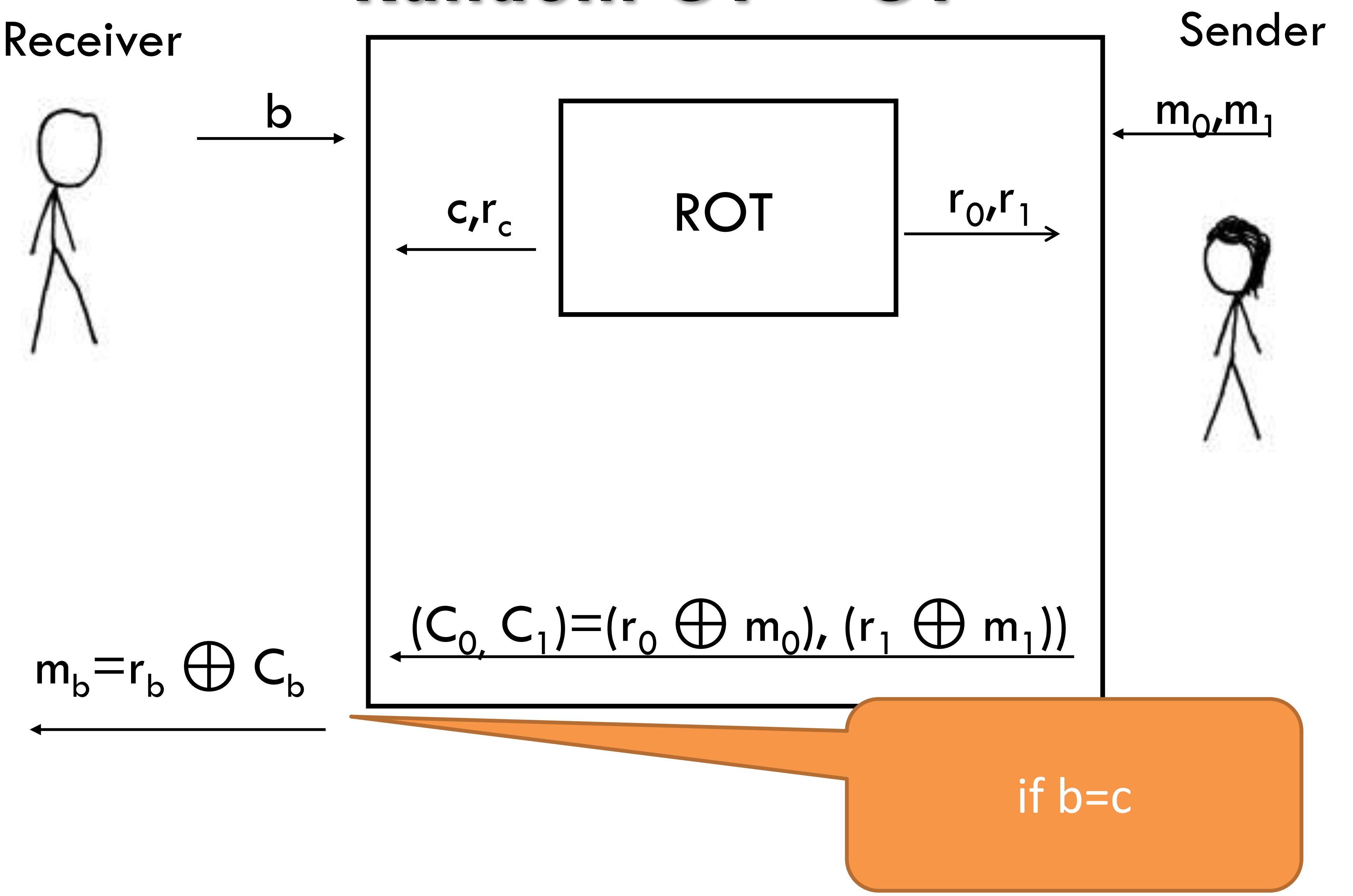
OT = shared AND



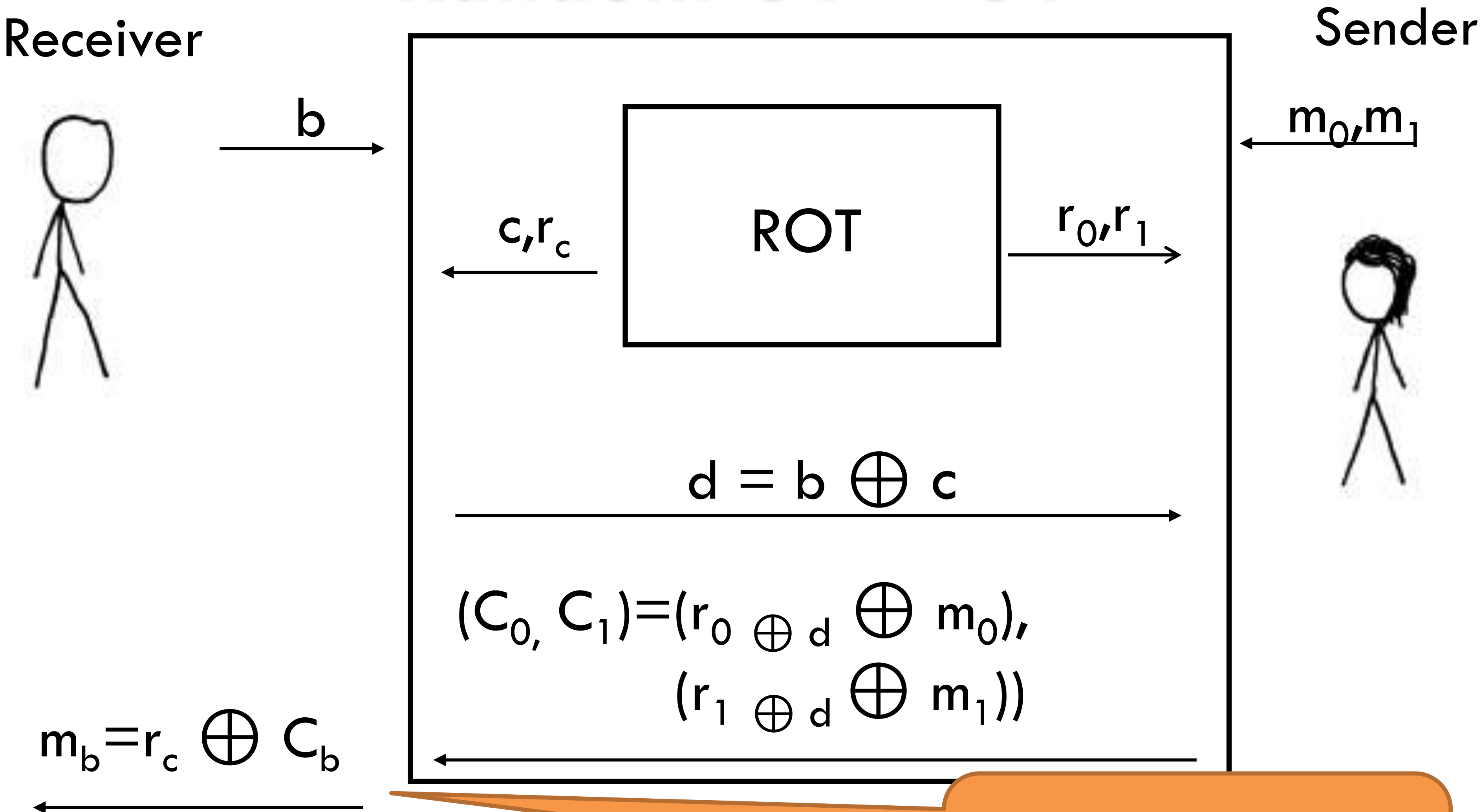
Stretching OT



Random OT = OT

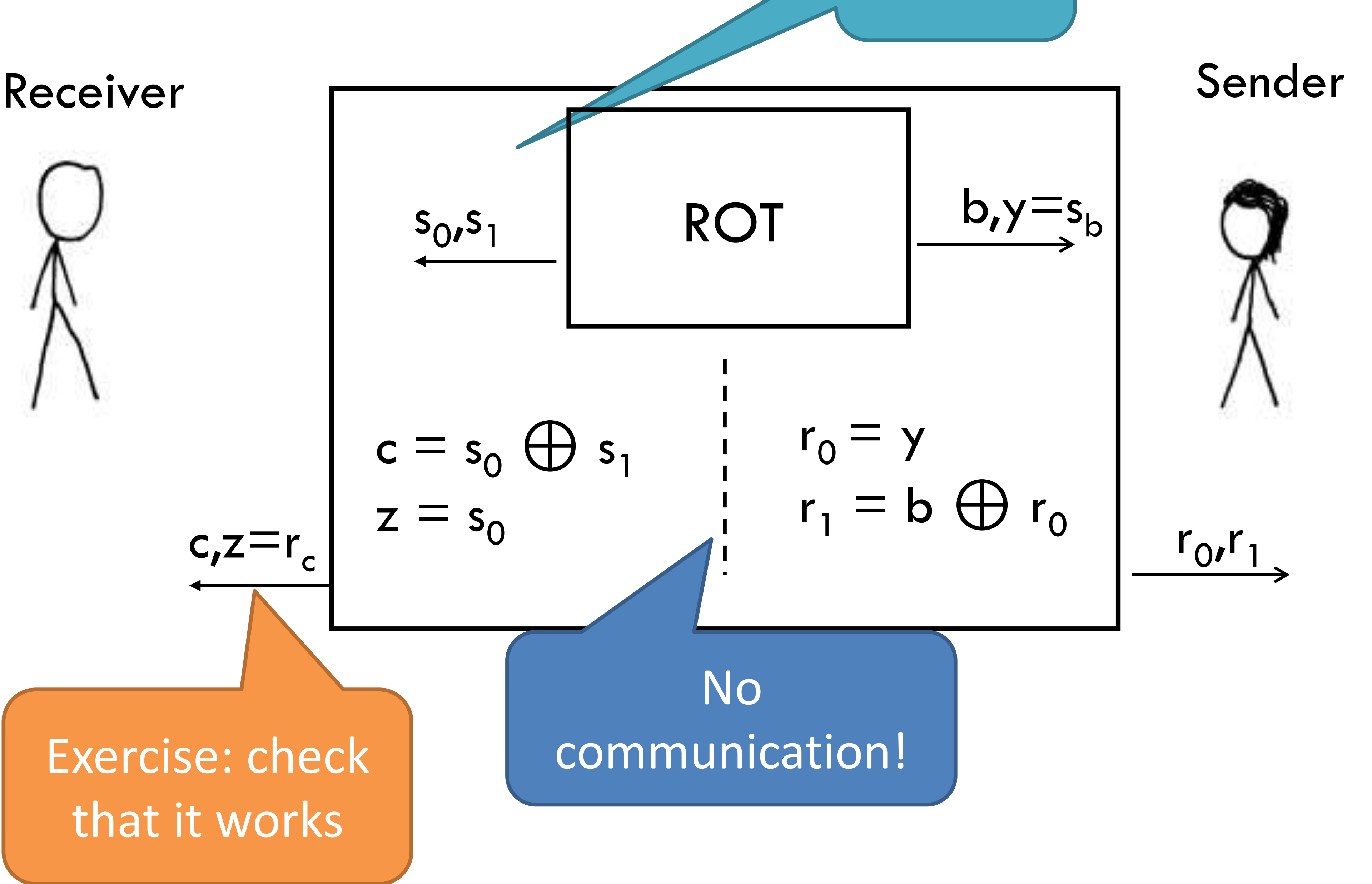

Receiver
Sender
b
m_0, m_1
ROT
c, r_c
r_0, r_1
$(C_0, C_1) = (r_0 \oplus m_0), (r_1 \oplus m_1))$
$m_b = r_b \oplus C_b$
if b=c

Random OT = OT



Exercise: check that it works!

(R)OT is symmetric

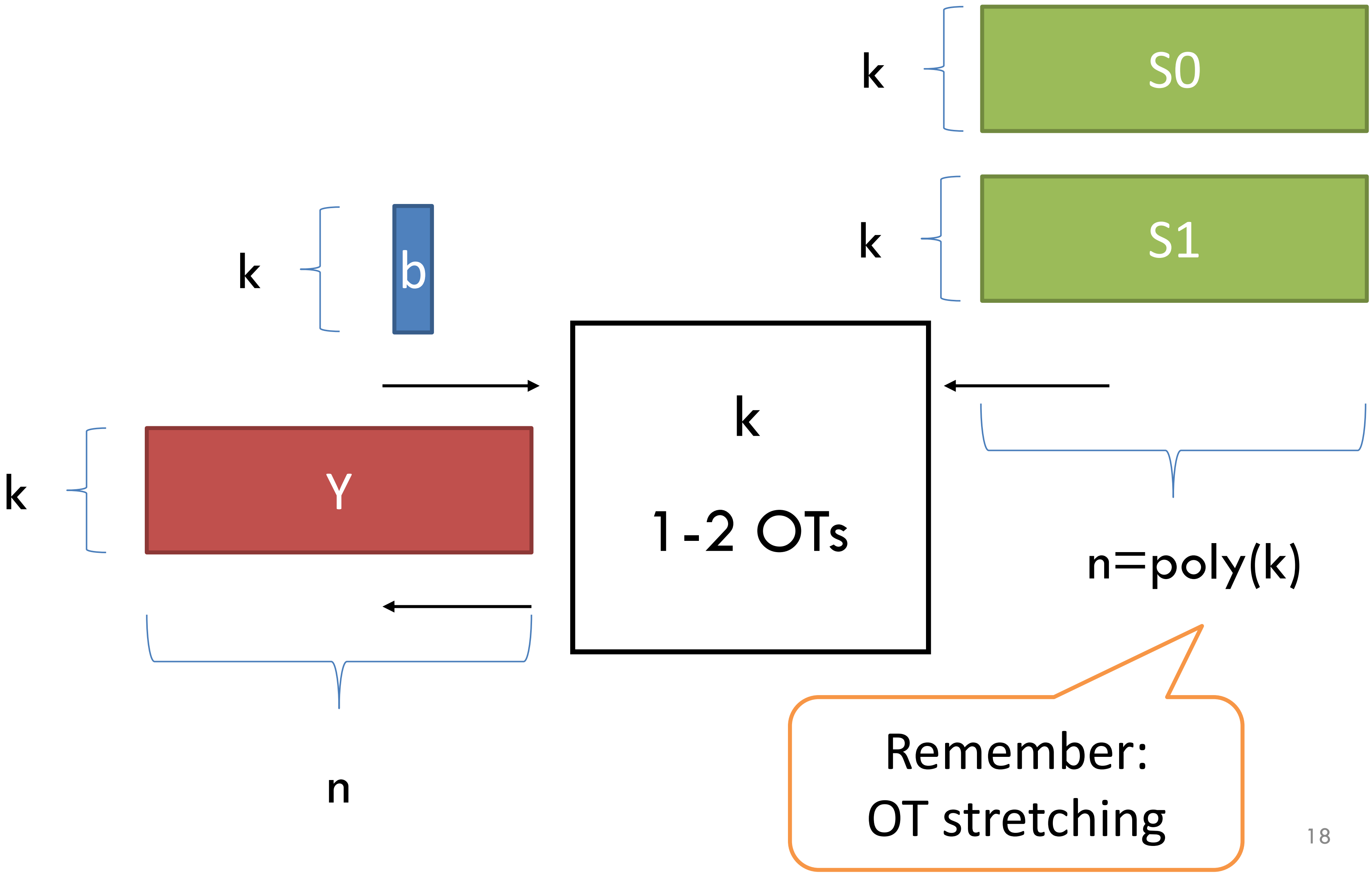


OT Extension (passive security)

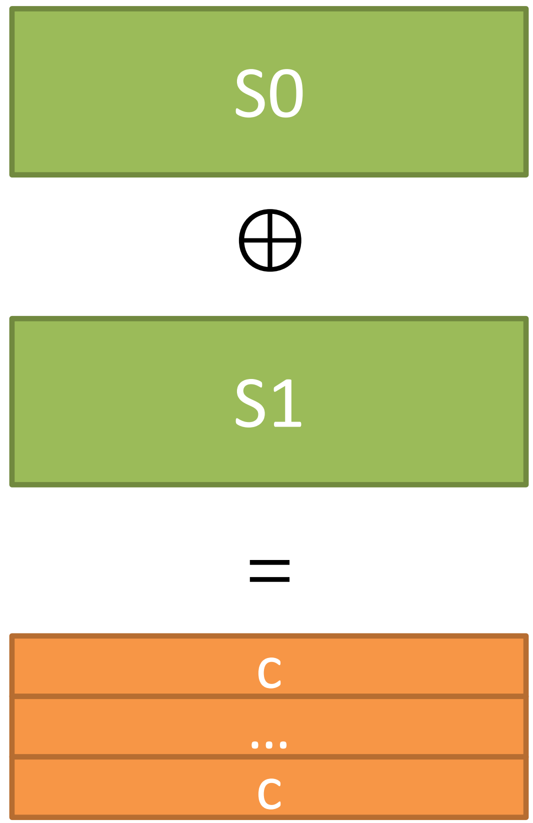
OT Extension

- OT provably requires public-key primitivies.
- OT extension is the OT analogue of hybrid encryption
  - *(RSA(pk,k), AES(k,m))*
- **Start from k "real" OTs → Extend them to poly(k) OTs using only few symmetric primitives (PRG/hash function) per generated OT**

OT Extension, Pictorially

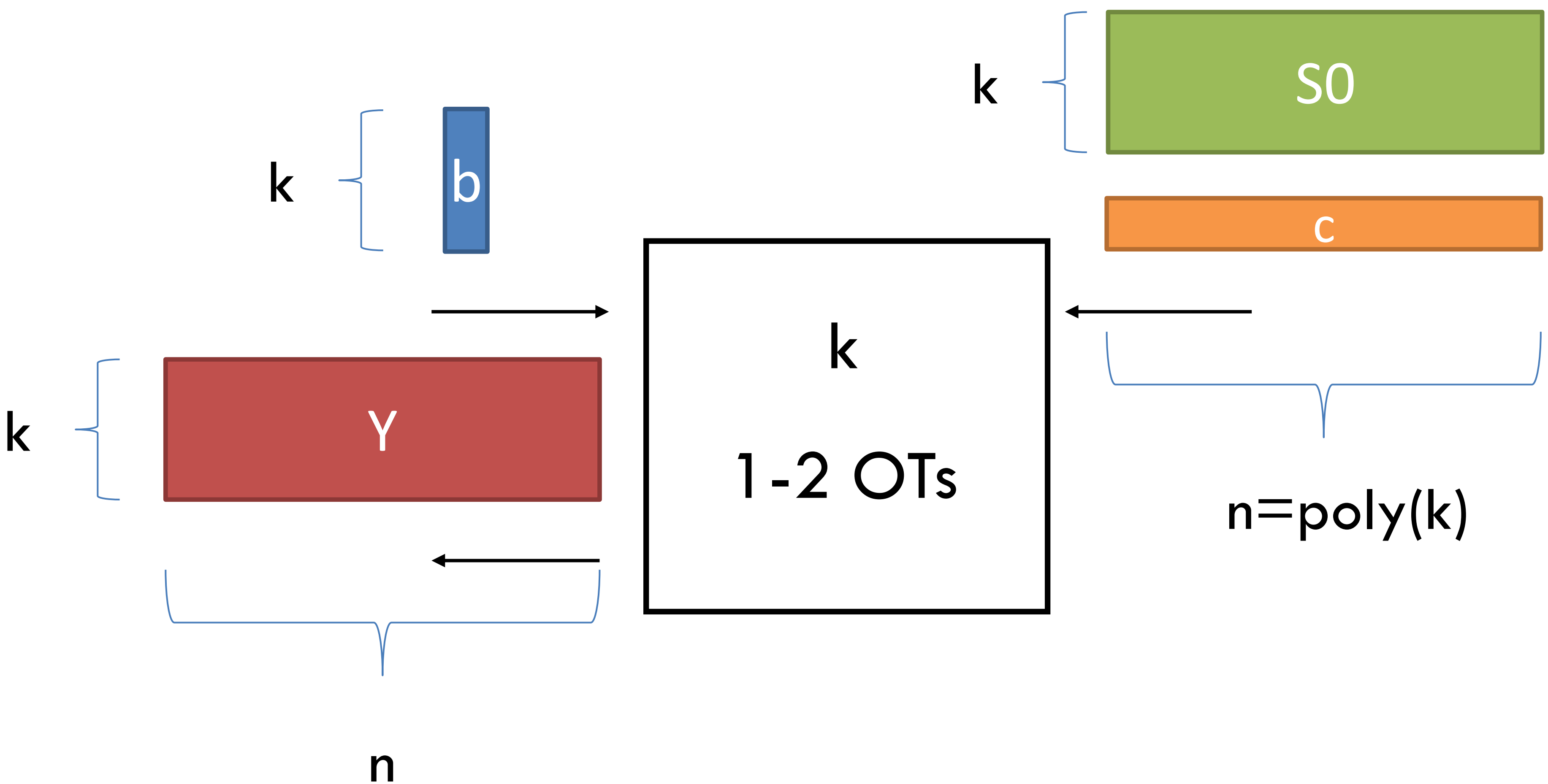


Condition for OT extension

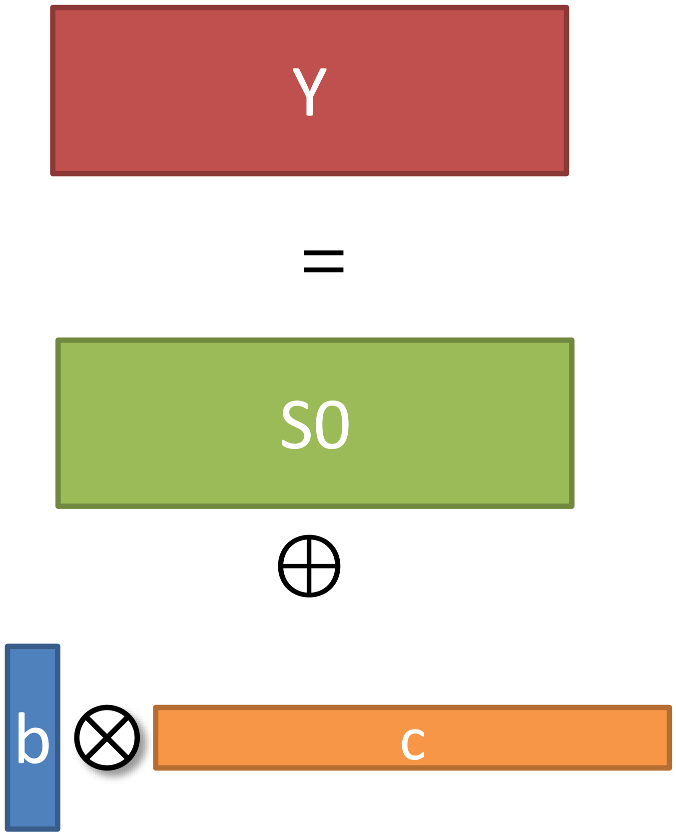


Problem for active security!

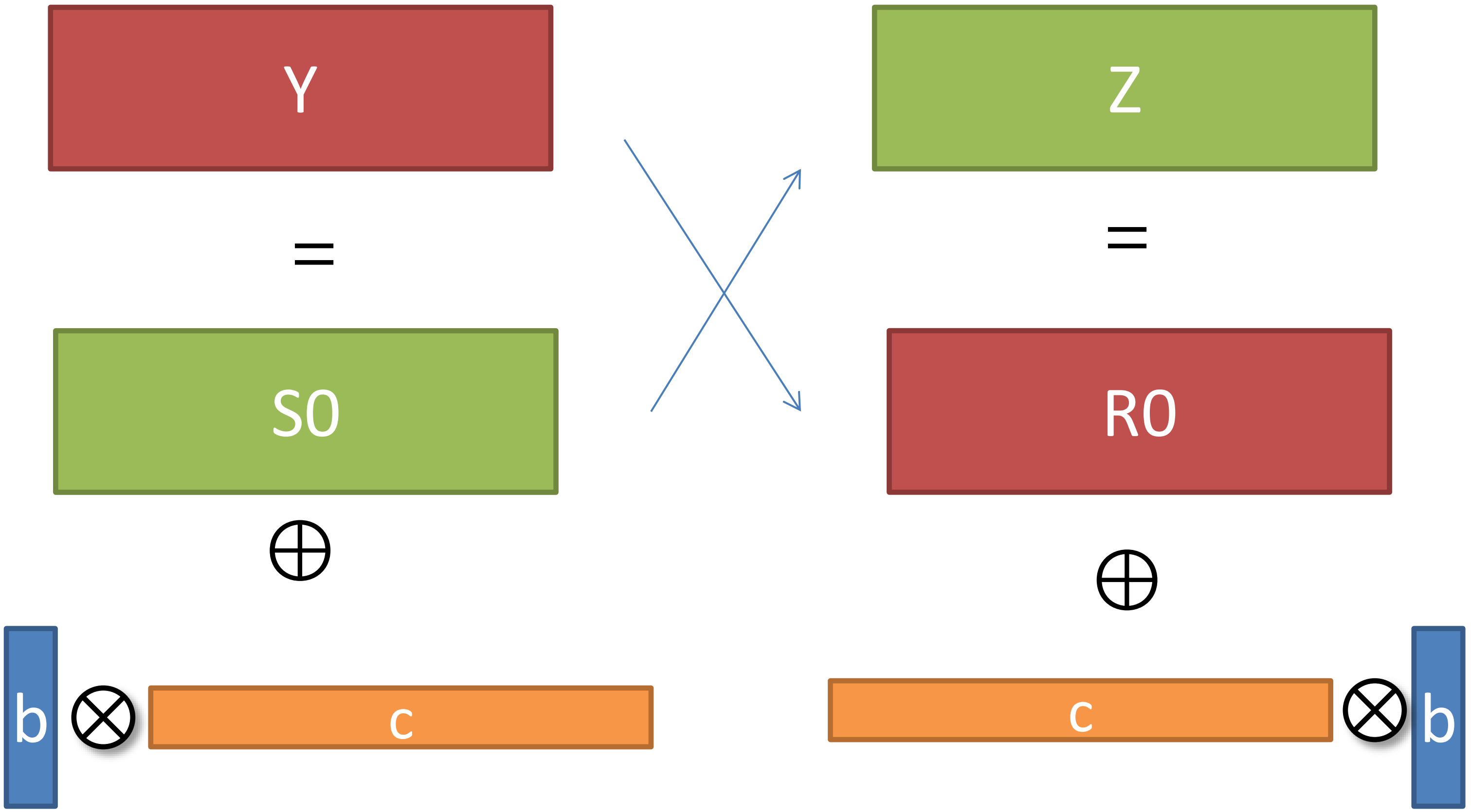
OT Extension, Pictorially



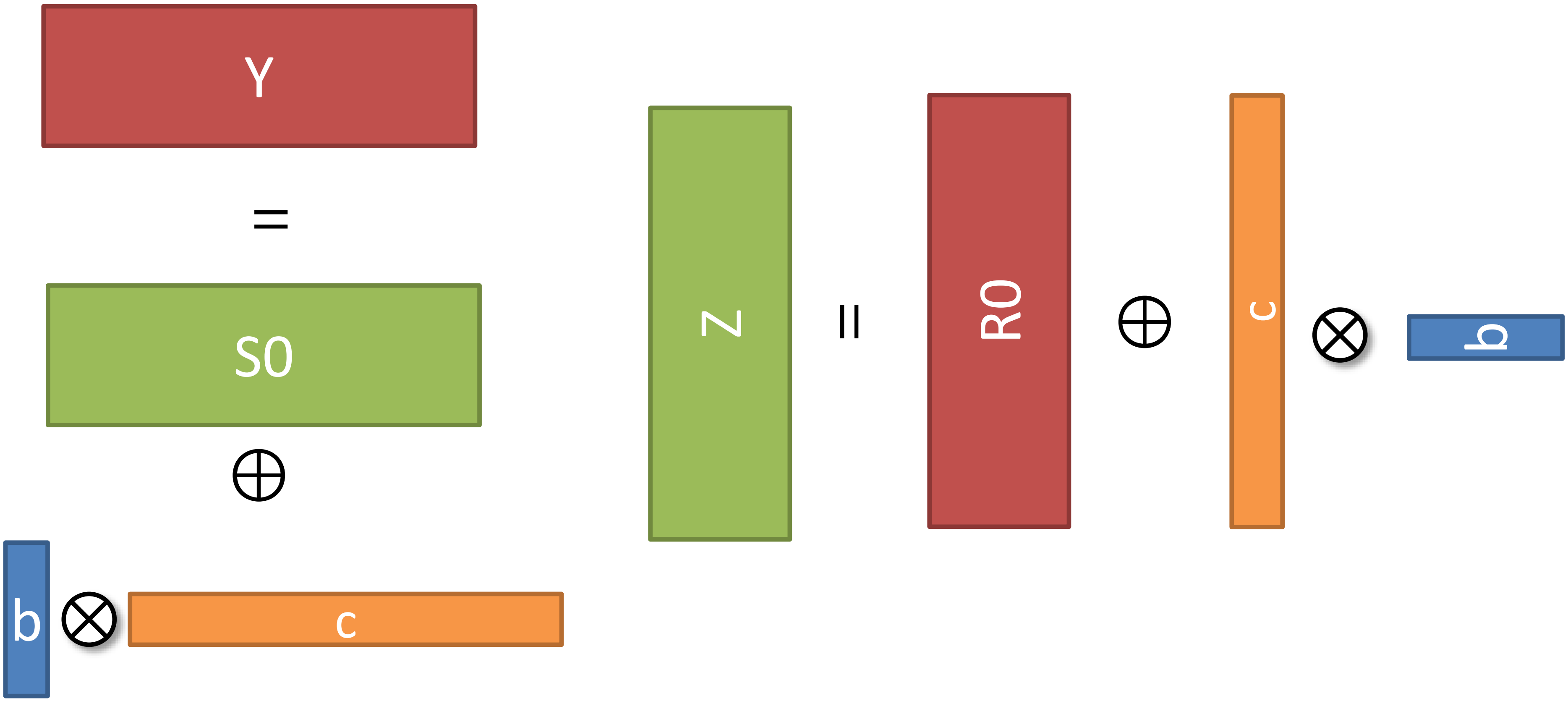
OT Extension, Pictorially



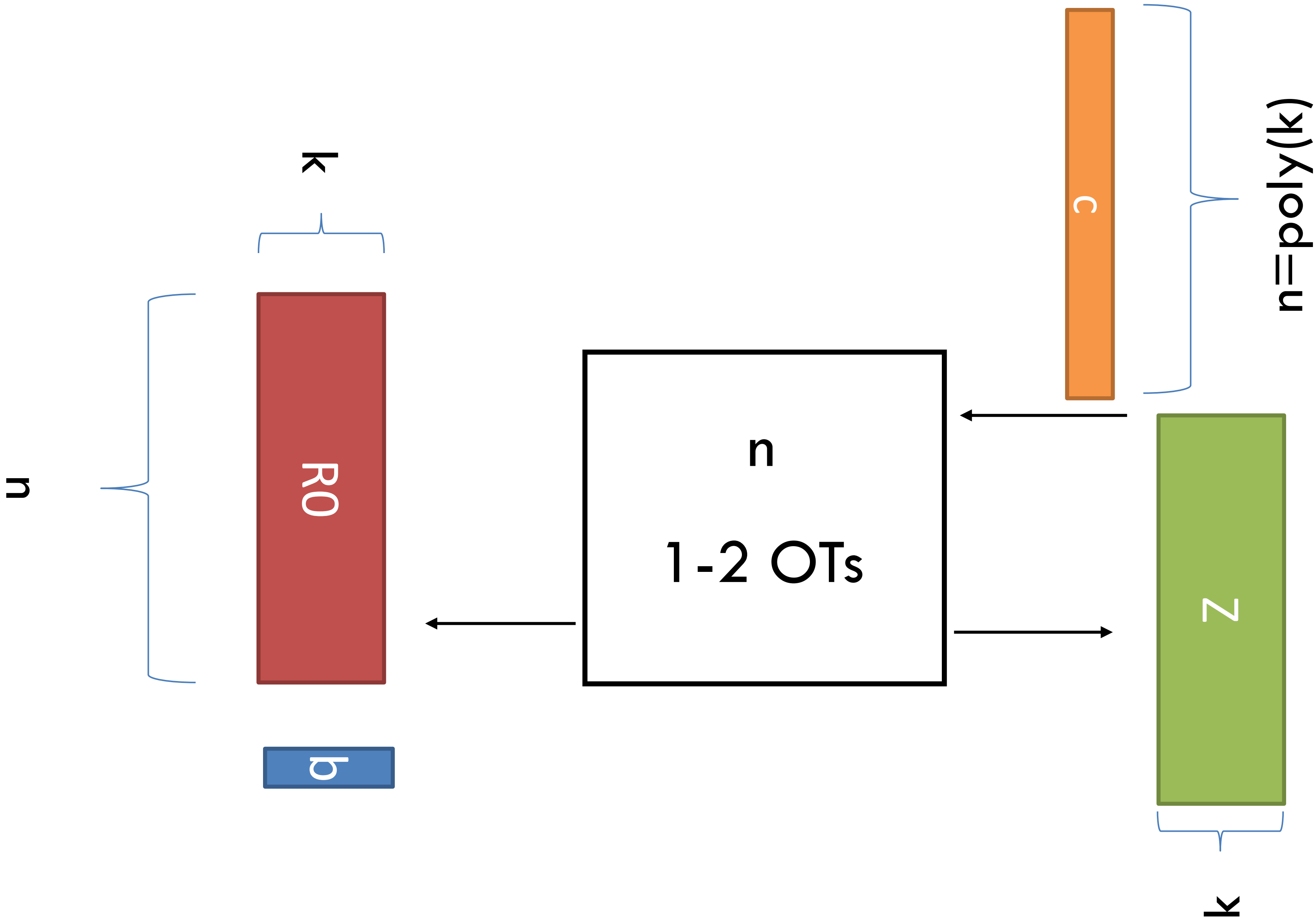
OT Extension, Turn your head!



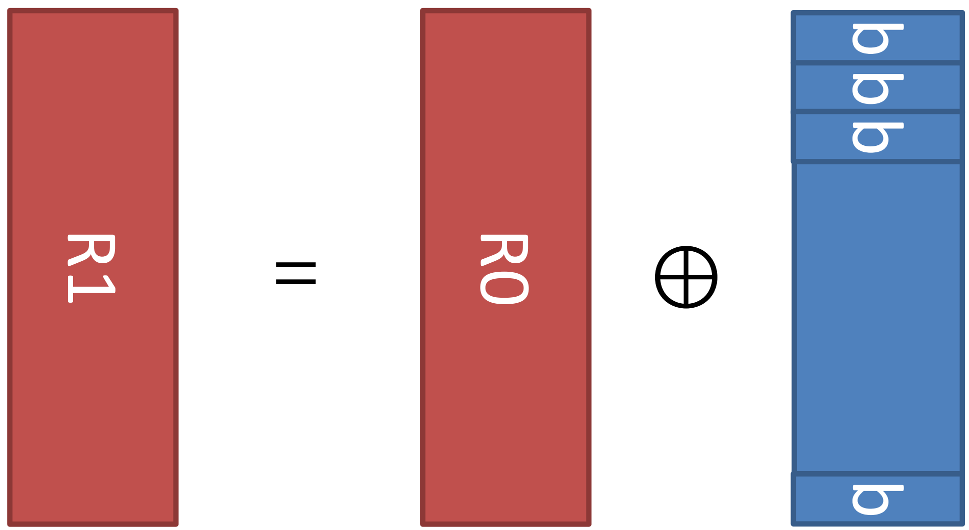
OT Extension, Turn your head!



OT Extension, Pictorially



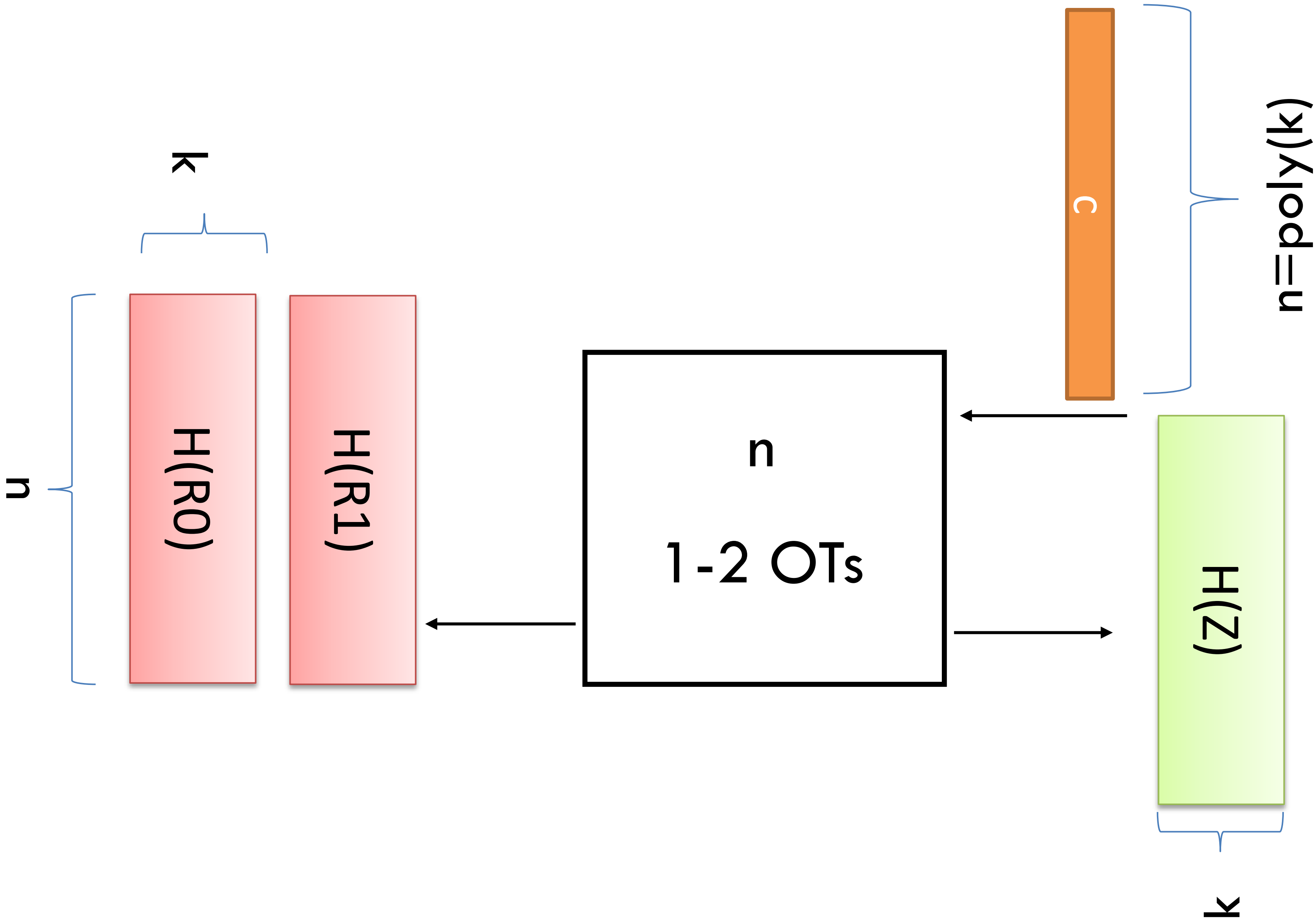
Defining R1



Finishing UP

- R0, R1 are not really random (strongly correlated)
- **Compute** (for each of the n rows)
  - T0 =H(R0), T1=H(R1)
- Using a **correlation robust hash function** H s.t.
  - $\{X_0,\ldots,X_n,H(X_0 \oplus R),\ldots,H(X_n \oplus R)\}$
  - $\{X_0,\ldots,X_n,Y_0,\ldots,Y_n\}$ // (X_i's,Y_i's random)
- Are computationally **indistinguishable**

OT Extension, Pictorially



Recap

1. Start with **k 1-2 OT for strings of length k**
2. **Stretch them** to length n=poly(k) using a PRG
3. Set each pair of messages s^i_0, s^i_1 s.t., $\mathbf{s^i_0 \oplus s^i_1 = c}$
4. **Turn your head** (s/r swap roles)
5. The bits of c are the new **choice bits**
6. The new messages are of the form $\mathbf{r^j_0, r^j_1 = r^j_0 \oplus b}$
7. Break the correlation: $\mathbf{t^j_0 = H(r^j_0), t^j_1 = H(r^j_1)}$

- **Not secure against active adversaries**

How to construct OT

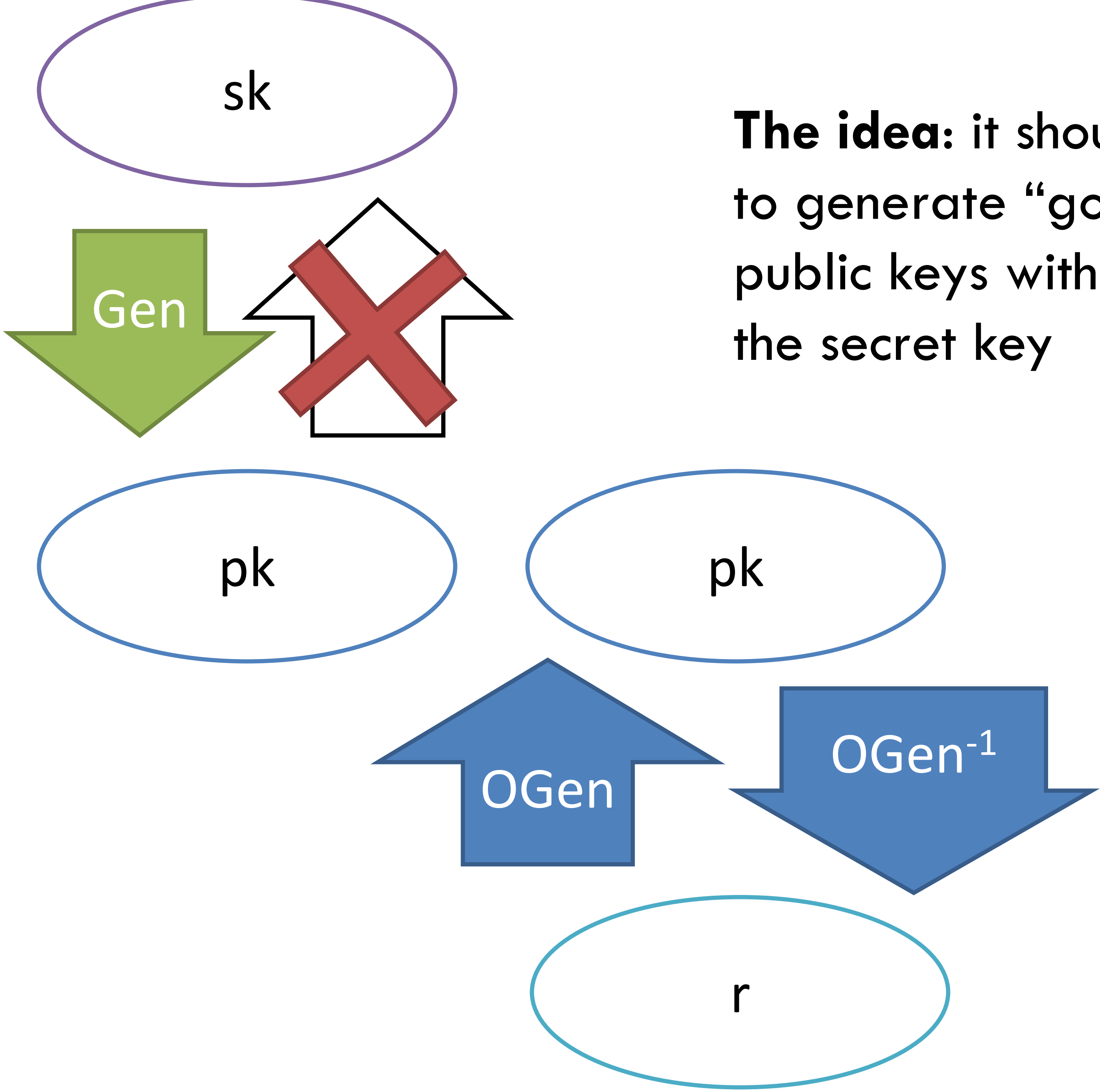
OT from PKE+

- Take a PKE (Gen,Enc,Dec)
  - Gen → (pk,sk)
  - Enc(pk,m) → C
  - Dec(sk,C) → m
  - **"IND-CPA Security"**

$$(pk,Enc(pk,m)) \sim (pk,Enc(pk,r))$$

- Augmeneted with (OGen, $OGen^{-1}$)
  - OGen(r) → pk
  - OGen(pk) → r

$$pk^{OGen} \sim pk^{Gen}$$



The idea: it should be possible to generate "good looking" public keys without learning the secret key

OT from PKE+

- **Receiver** (with choice bit b)
  - Compute pk_b,sk $\leftarrow$ Gen
  - Compute pk_{1-b} $\leftarrow$ Ogen
  - Send pk_0,pk_1
- **Sender** (with messages m_0 m_1)
  - Encrypt C_0=E(pk_0,m_0), C_1=E(pk_1,m_1)
- **Receiver**
  - m_b=Dec(sk,C_b)

- **What can go wrong?**

Discrete Log (additive notation)

- <G> group of order p (prime) generated by G
  - **DL**: given H it is hard to compute x s.t. xG=H
  - **CDH**: given (G,aG,bG) it is hard to compute abG
  - **DDH** given (G,aG,bG,cG) it is hard to decide if c=ab
- ElGamal Encryption
  - **Gen** → (G, H=xG)
  - **Enc**(pk,m)→ (rG, rH + m)
  - **Dec**(sk,C,D) → D – xC
- *In "many" groups it is possible to sample uniform H (without learning DL)*

Exercise: we do not know how to sample RSA public keys obliviously.
But ciphertexts can.
Can you build an OT protocol from RSA?

Naor Pinkas OT

- Sender
  - Choose random F
- Receiver (with choice bit b)
  - Pick $H_b = xG$
  - $H_{1-b} = F - H_b$
  - Send H_0
- Sender (with messages m_0 m_1)
  - Compute $H_1 = F - H_0$
  - Encrypt $Enc(H_0, m_0)$, $Enc(H_1, m_1)$

//idea: hard to compute DL of both H_0,H_1
(but cannot simulate)