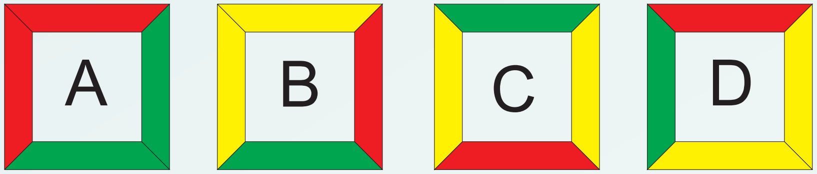
Lecture 4: CA and undecidability

- Reductions to cellular automata
- NW-determinism & one-dimensional CA
- Snakes and reversibility

Recall from yesterday



The **tiling problem** (or the **Domino problem**) of Wang tiles is the decision problem to determine if a given finite set of Wang tiles admits a valid tiling of the plane.

Theorem (R.Berger 1966): The tiling problem of Wang tiles is undecidable.

The tiling problem can be reduced to various decision problems concerning (two-dimensional) cellular automata

$\Longrightarrow$ undecidability of these problems

This is not so surprising since Wang tilings are **”static”** versions of **”dynamic”** cellular automata.

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- state set T,
- the von Neumann -neighborhood,
- the local update rule that keeps a tile unchanged if and only if its colors match with the neighboring tiles.

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Note: For one-dimensional CA it is easily decidable whether fixed points exist.

More interesting reduction: A CA is called **nilpotent** if all configurations eventually evolve into the quiescent (=all states in state q) configuration.

Observation: In a nilpotent CA all configurations must become quiescent within a bounded time, that is, there is uniform bound n such that $G^n(c)$ is quiescent, for all $c \in S^{\mathbb{Z}^d}$.

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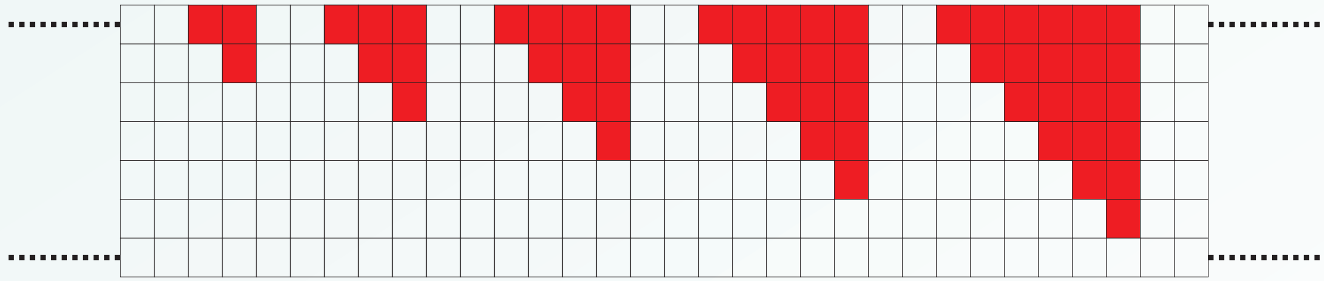
Proof: Suppose contrary: for every n there is a configuration c_n such that $G^n(c_n)$ is not quiescent. Then c_n contains a finite pattern p_n that evolves in n steps into some non-quiescent state. A configuration c that contains a copy of every p_n never becomes quiescent, contradicting nilpotency.



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Proof: For any given set T of Wang tiles we construct a two-dimensional CA that is nilpotent if and only if T does not admit a tiling.

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- State set is $S = T \cup \{q\}$ where q is a new symbol $q \notin T$,

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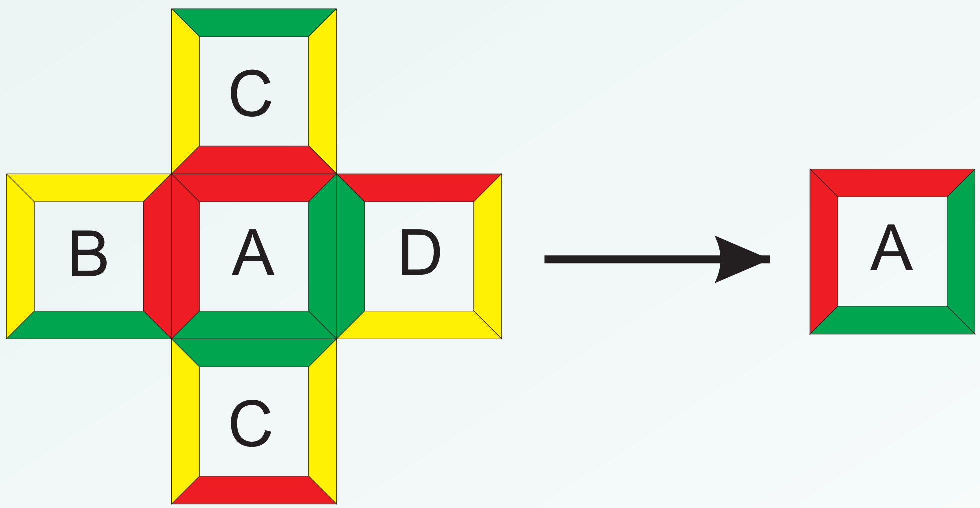

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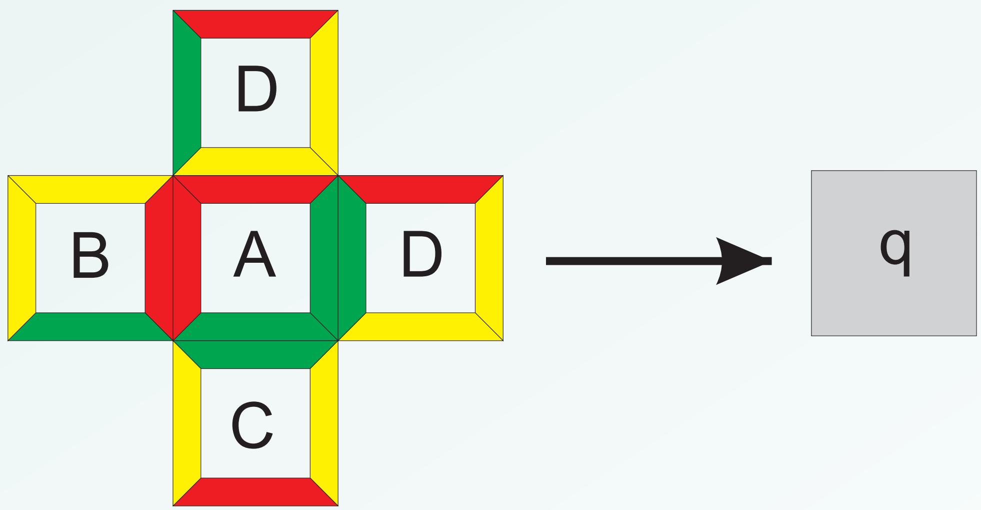

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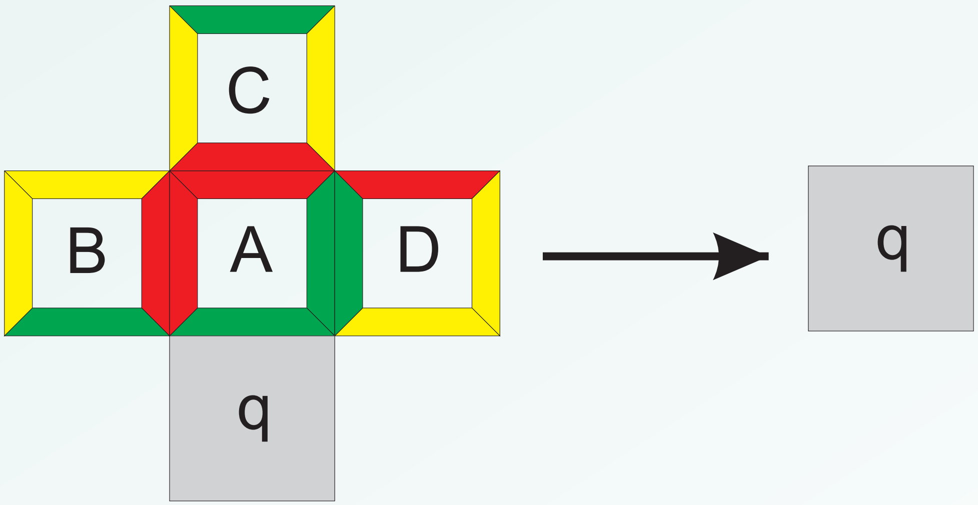

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$\Longleftarrow$ If T does not admit a valid tiling then every $n \times n$ square contains a tiling error, for some n. State q propagates, so in at most $2n$ steps all cells are in state q. The CA is nilpotent. $\square$

If we do the previous construction for an aperiodic tile set T we obtain a two-dimensional CA in which

- every periodic configuration becomes eventually quiescent, but
- there are some non-periodic fixed points.

NW-deterministic tiles

Tilings relate naturally to two-dimensional CA.

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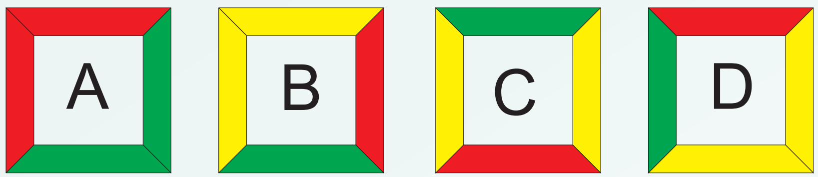
What about **one-dimensional CA** ?

We can strengthen Berger's result so that the **nilpotency** can be proved undecidable for one-dimensional CA as well.

The basic idea is to view **space-time diagrams** of one-dimensional CA as tilings.

Tile set T is **NW-deterministic** if no two tiles have identical colors on their top edges and on their left edges. In a valid tiling the left and the top neighbor of a tile uniquely determine the tile.

For example, our sample tile set



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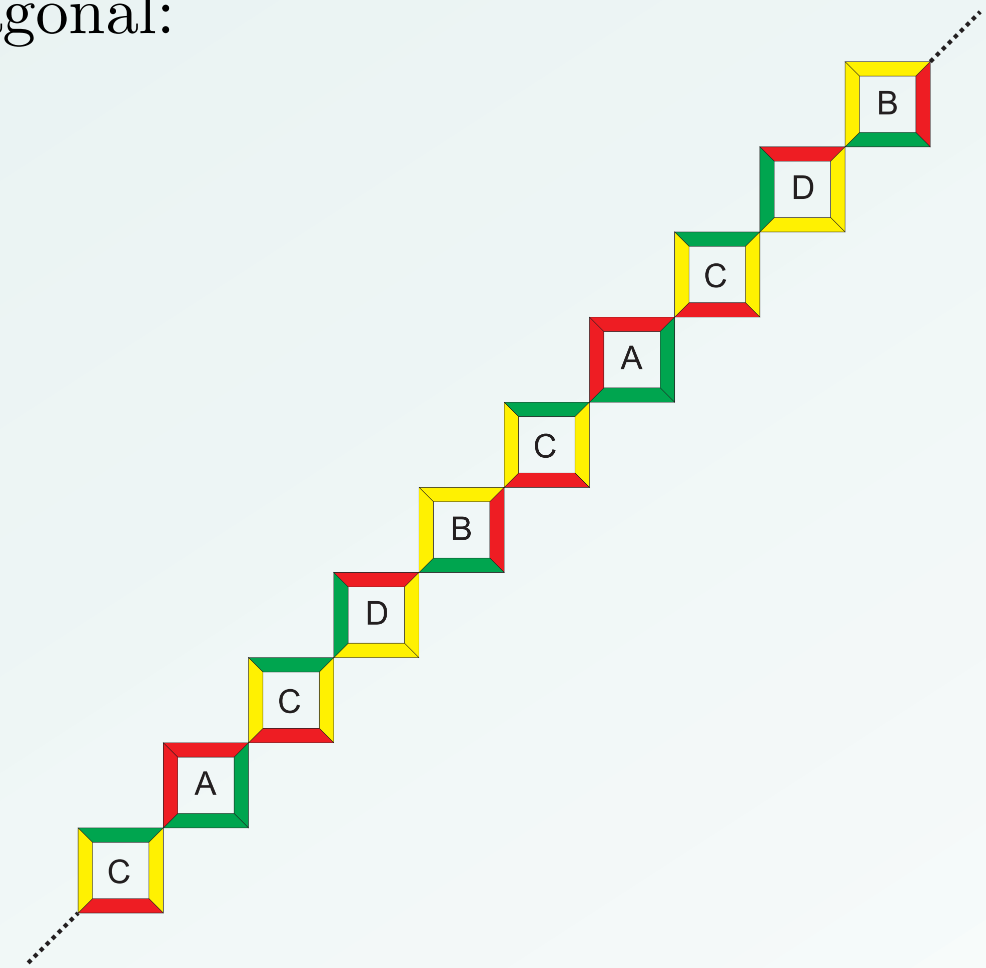
In any valid tiling by NW-deterministic tiles, NE-to-SW diagonals uniquely determine the next diagonal below them. The tiles of the next diagonal are determined locally from the previous diagonal:



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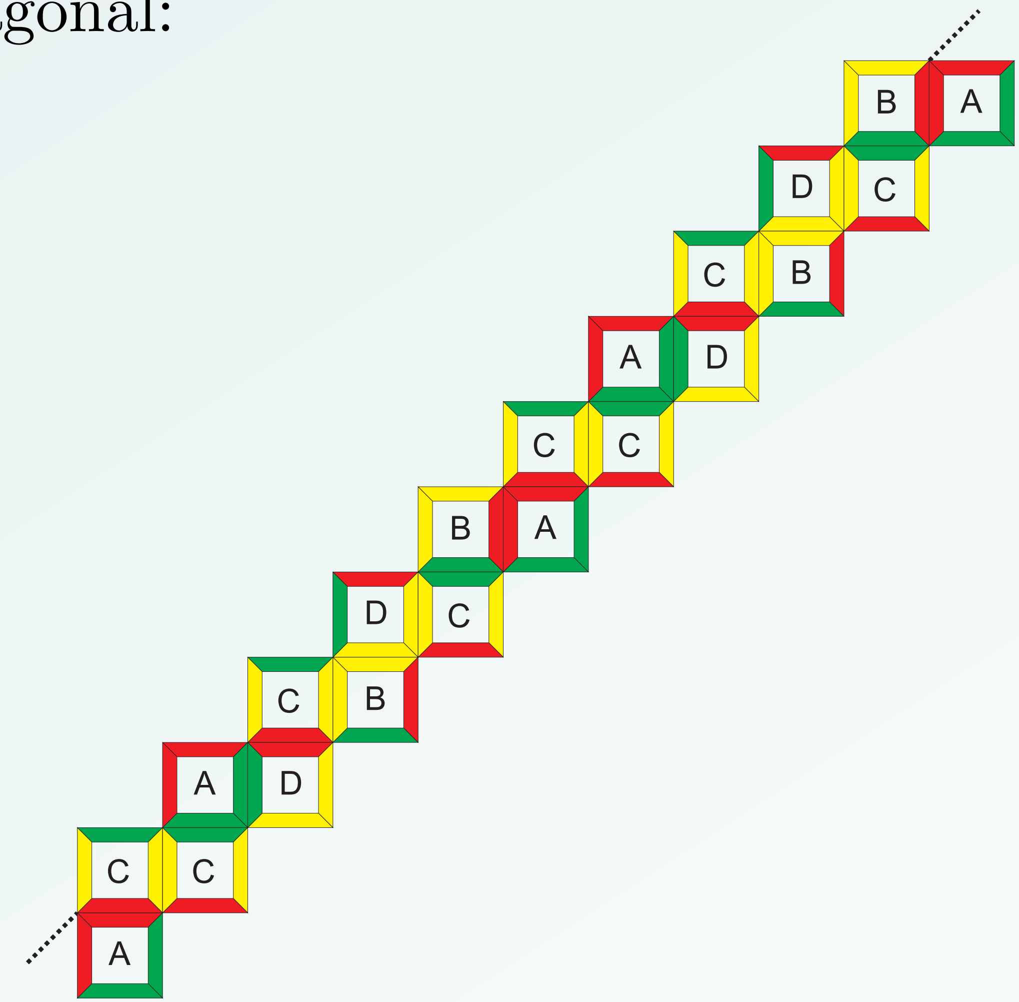

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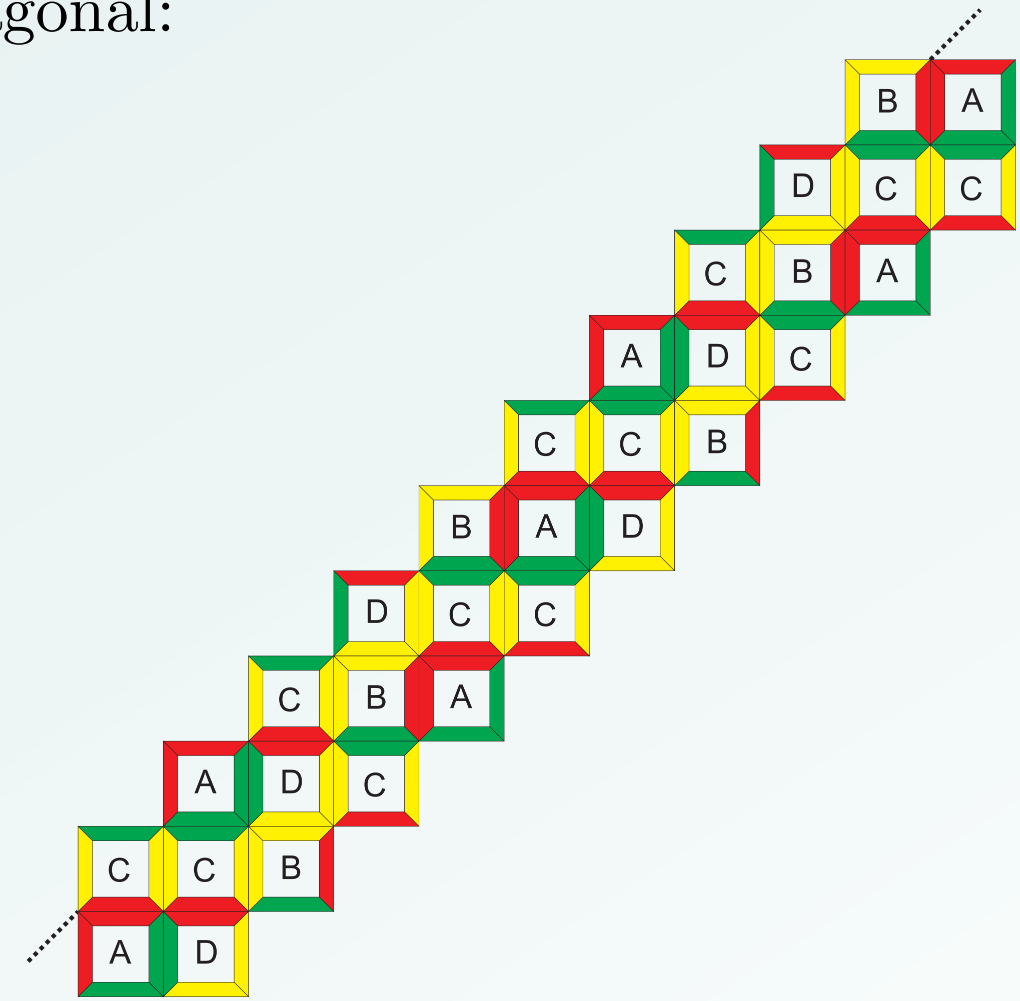

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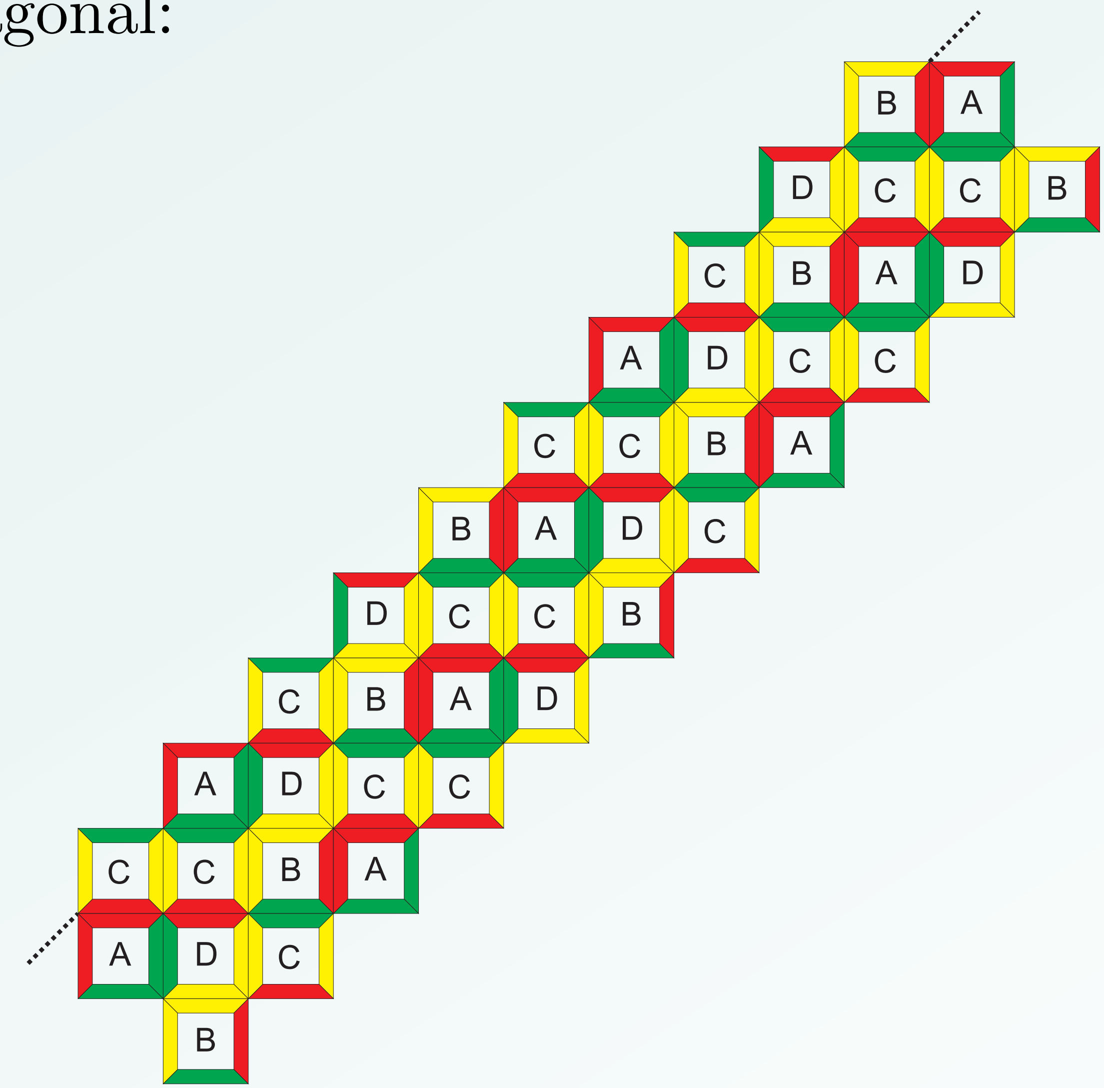

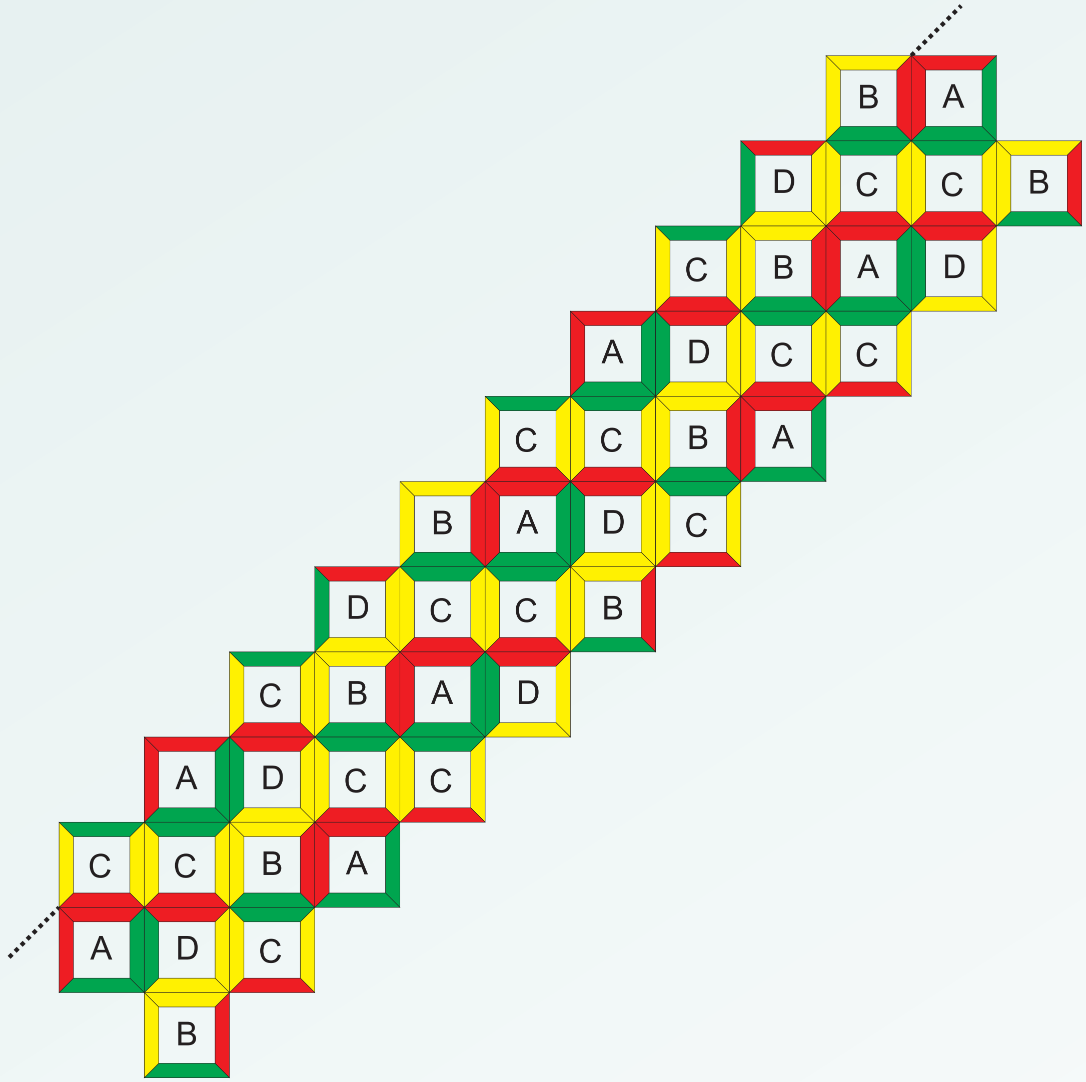

If diagonals are interpreted as configurations of a one-dimensional CA, valid tilings represent space-time diagrams.

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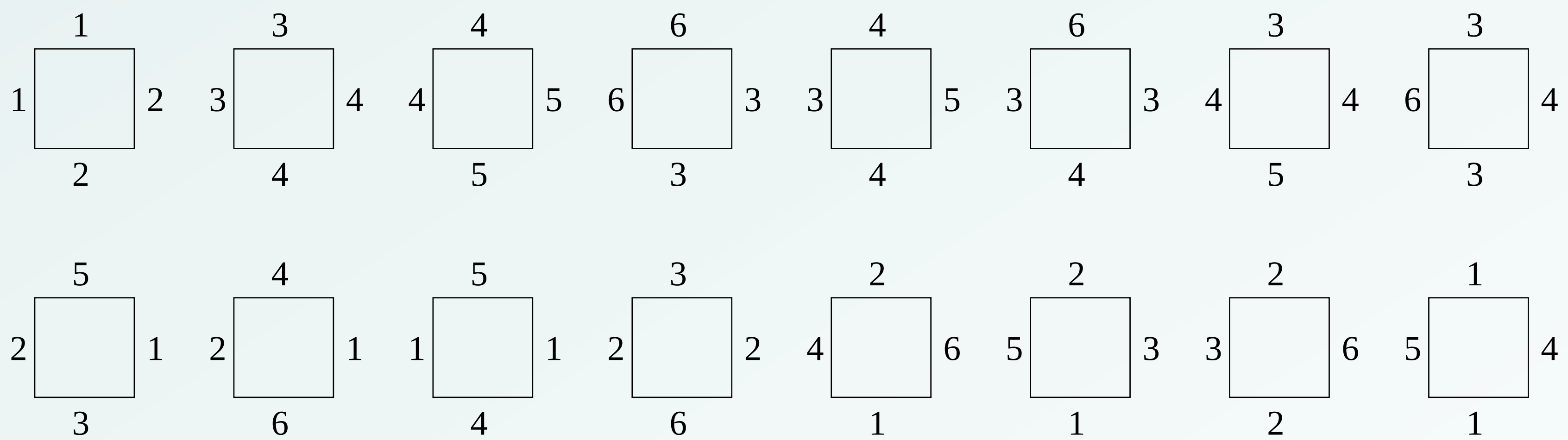
YES!

1. There are **aperiodic** NW-deterministic tiles sets:



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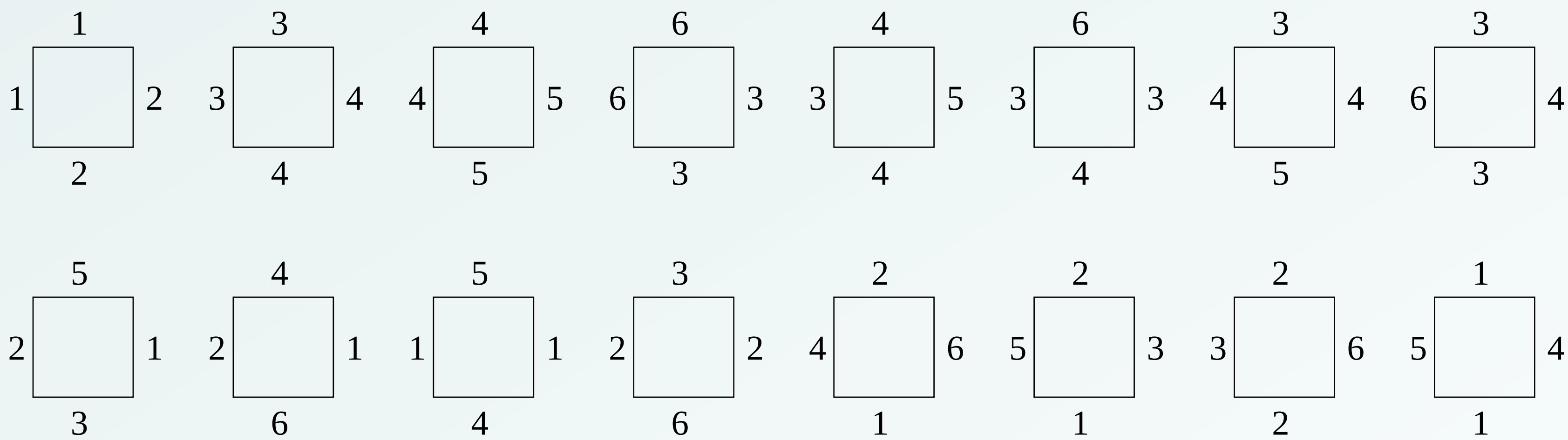

Amman's 16 tile aperiodic tile set

2. With a bit of effort (proof omitted):

Theorem: The tiling problem is undecidable among NW-deterministic tile sets.

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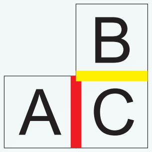
- **state set** is $S = T \cup \{q\}$ where q is a new symbol $q \notin T$,
- **neighborhood** is $(0, 1)$,
- **local rule** $f : S^2 \longrightarrow S$ is defined as follows:
  - $f(A, B) = C$ if the colors match in 

  - $f(A, B) = q$ if $A = q$ or $B = q$ or no matching tile C exists.

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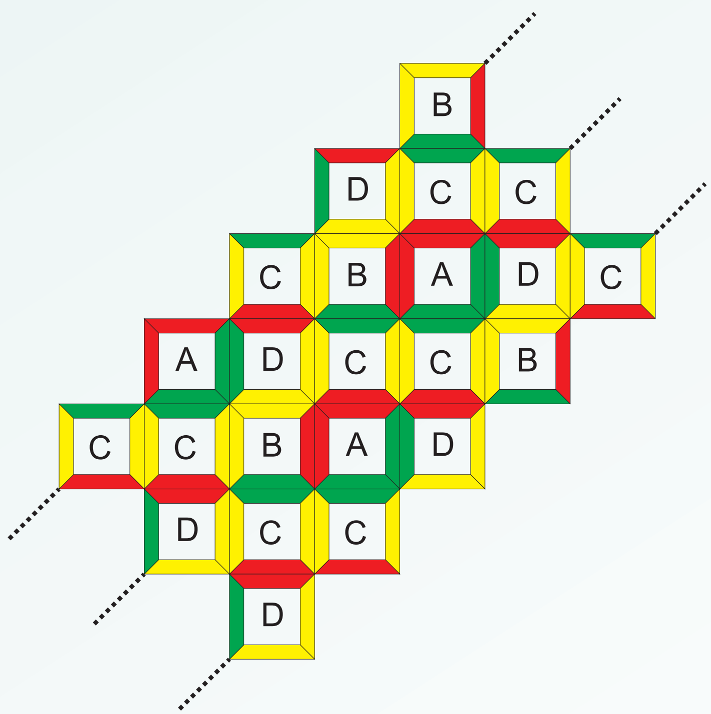
Proof:

$\Longrightarrow$ If T **admits a tiling** c then diagonals of c are configurations that never evolve into the quiescent configuration. So the CA is **not nilpotent**.



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$\Longleftarrow$ If T **does not admit a tiling** then every $n \times n$ square contains a tiling error, for some n. Hence state q is created inside every segment of length n.

Since q spreads, the whole configuration becomes eventually quiescent. The CA is **nilpotent**.

The tiling problem is undecidable for NW-deterministic tile sets, so

Theorem: It is undecidable whether a given one-dimensional CA is nilpotent. □

The tiling problem is undecidable for NW-deterministic tile sets, so

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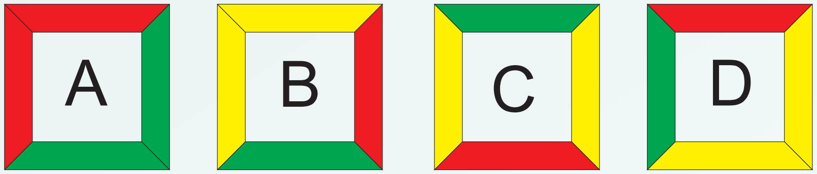
If we do the previous construction using an aperiodic set then we have an interesting one-dimensional CA:

- all periodic configurations eventually die, but
- there are non-periodic configurations that never create a quiescent state in any cell.

Analogously we can define NE-, SW- and SE-determinism of tile sets. A tile set is called **4-way** deterministic if it is deterministic in all four corners.

Our sample tile set is 4-way deterministic



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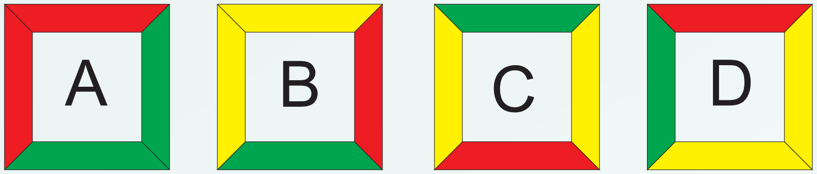


V.Lukkarila showed the following:

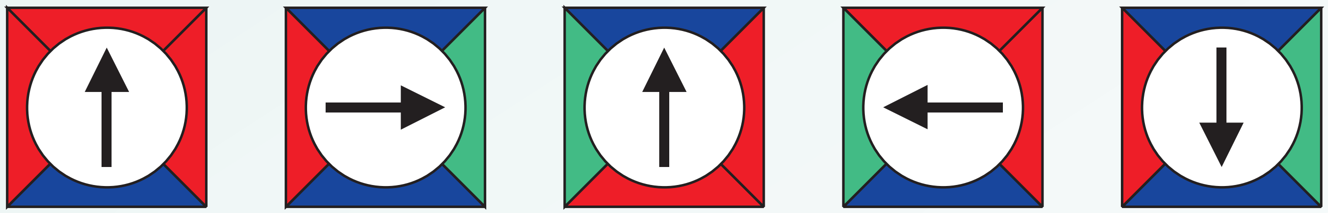
Theorem: The tiling problem is undecidable among 4-way deterministic tile sets.

This result provides some undecidability results for dynamics of **reversible** one-dimensional CA.

Snakes

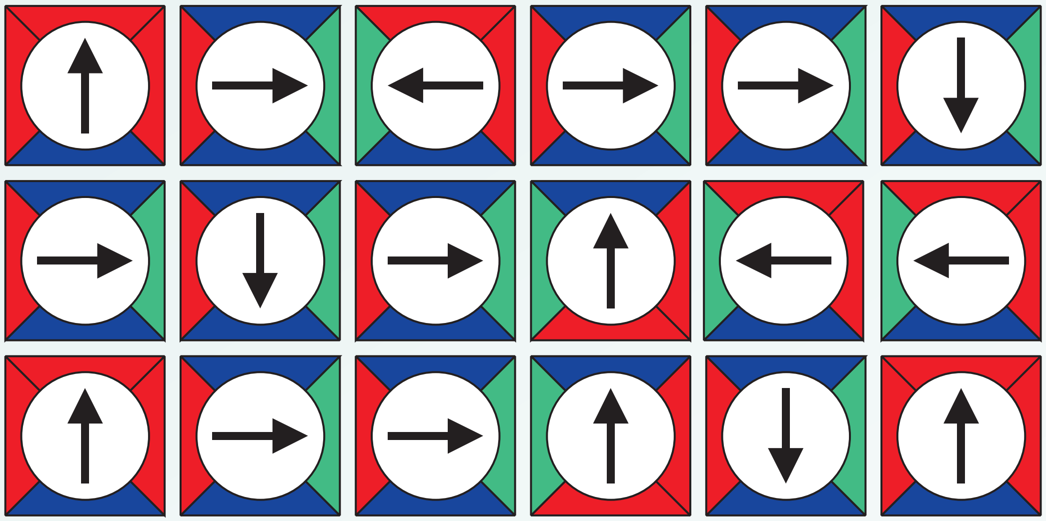
Snakes is a tile set with some interesting (and useful) properties.

In addition to colored edges, these tiles also have an arrow printed on them. The arrow is horizontal or vertical and it points to one of the four neighbors of the tile:



Such tiles with arrows are called **directed tiles**.

Given a configuration (valid tiling or not!) and a starting position, the arrows specify a path on the plane. Each position is followed by the neighboring position indicated by the arrow of the tile:



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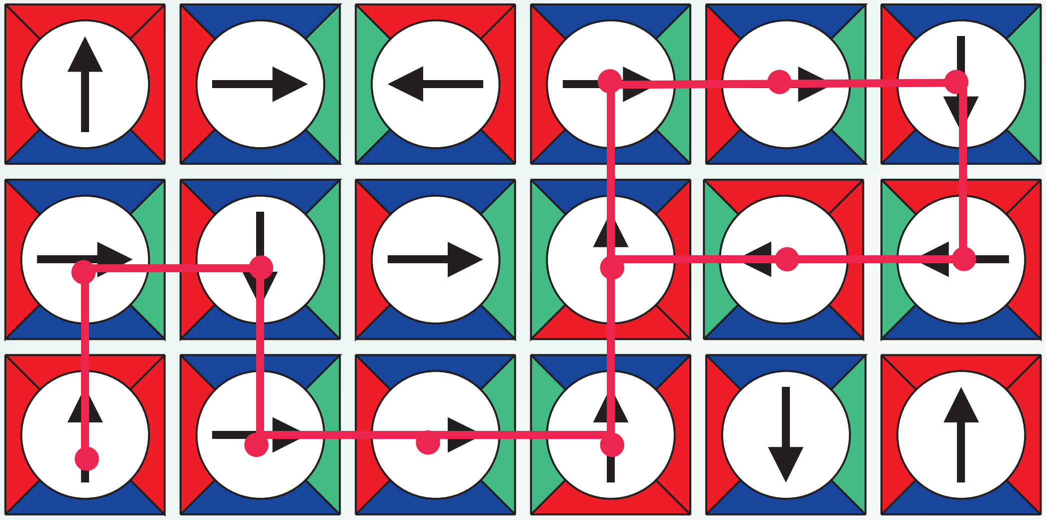

The path may enter a loop…

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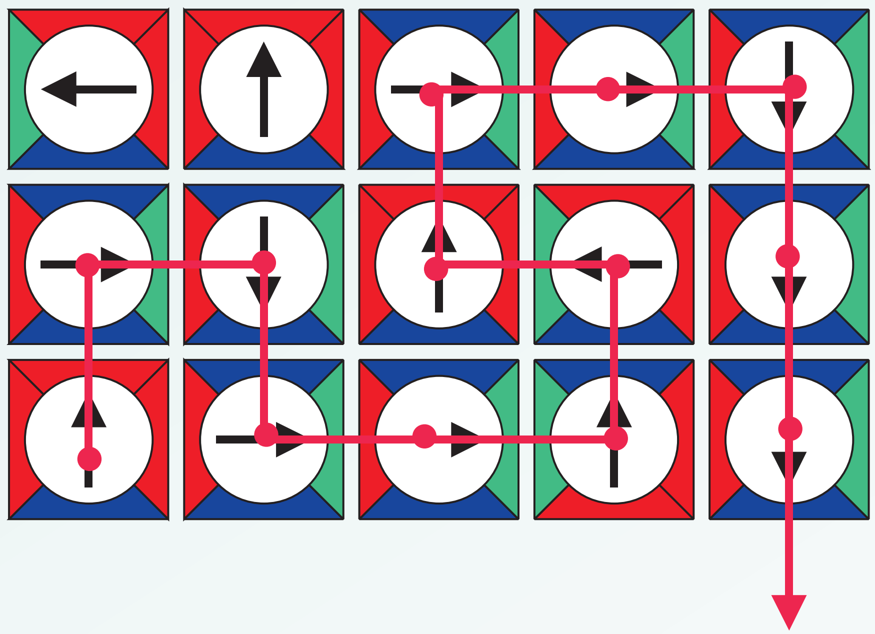
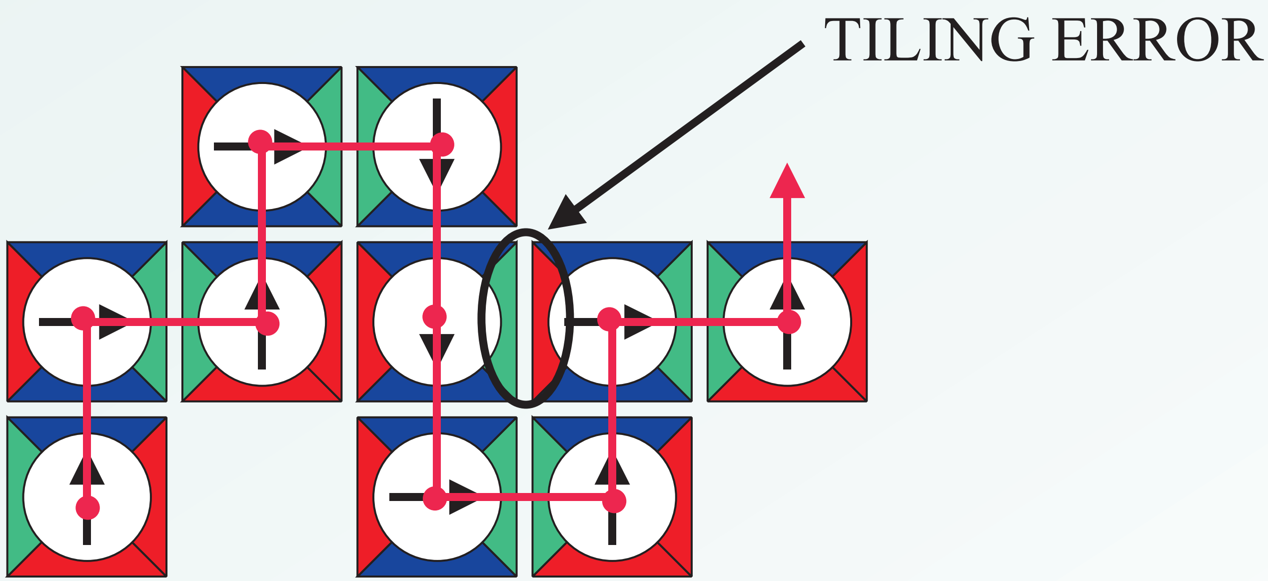


. . . or the path may be infinite and never return to a tile visited before.

The directed tile set SNAKES has the following property: On any configuration (valid tiling or not) and on any path that follows the arrows one of the following two things happens:

(1) Either there is a tiling error between two tiles both of which are on the path,



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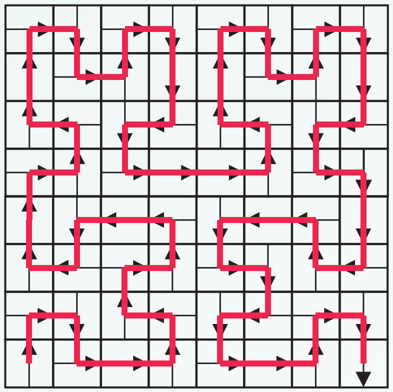
(1) Either there is a tiling error between two tiles both of which are on the path,

(2) or the path is a plane-filling path, that is, for every positive integer n there exists an $n \times n$ square all of whose positions are visited by the path.



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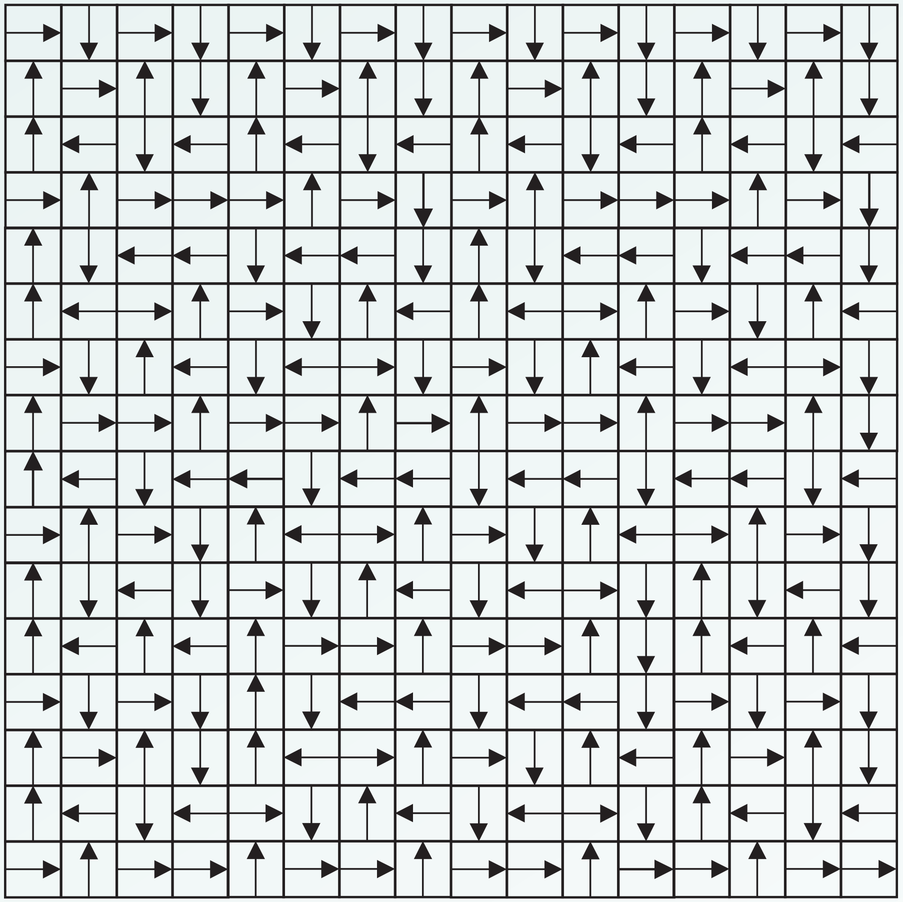
Note that the tiling may be invalid outside path P, yet the path is forced to snake through larger and larger squares.

SNAKES also has the property that it admits a valid tiling.

The paths that SNAKES forces when no tiling error is encountered have the shape of the well known plane-filling Hilbert-curve



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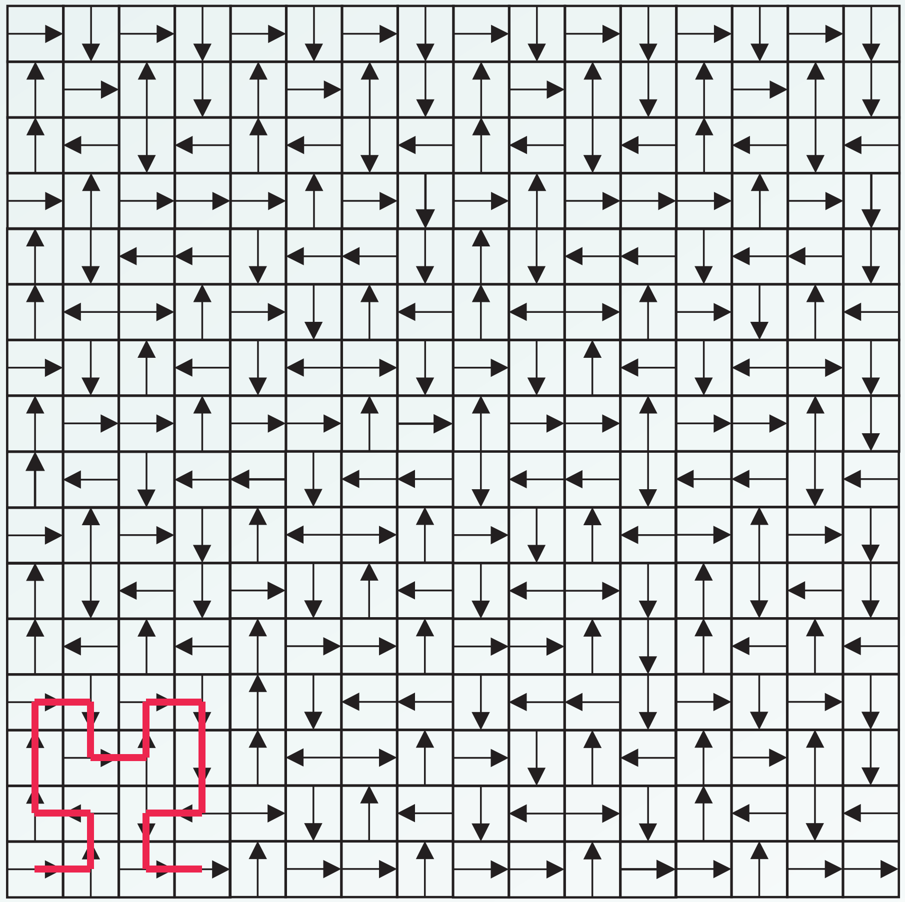

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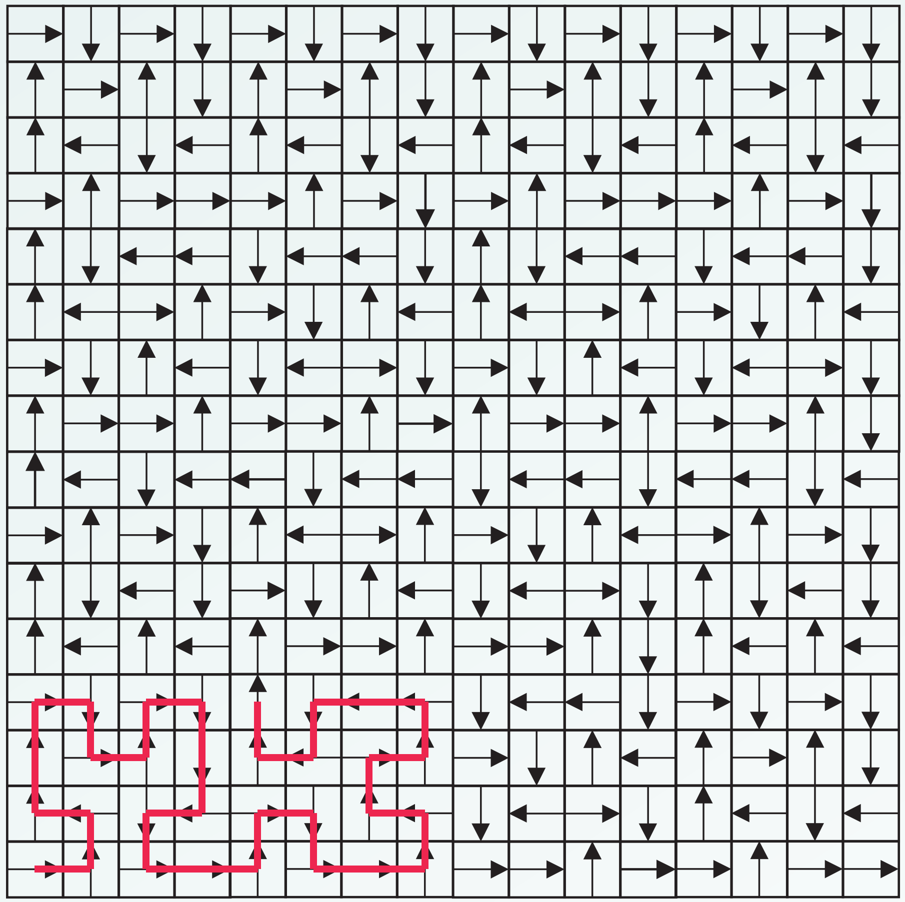

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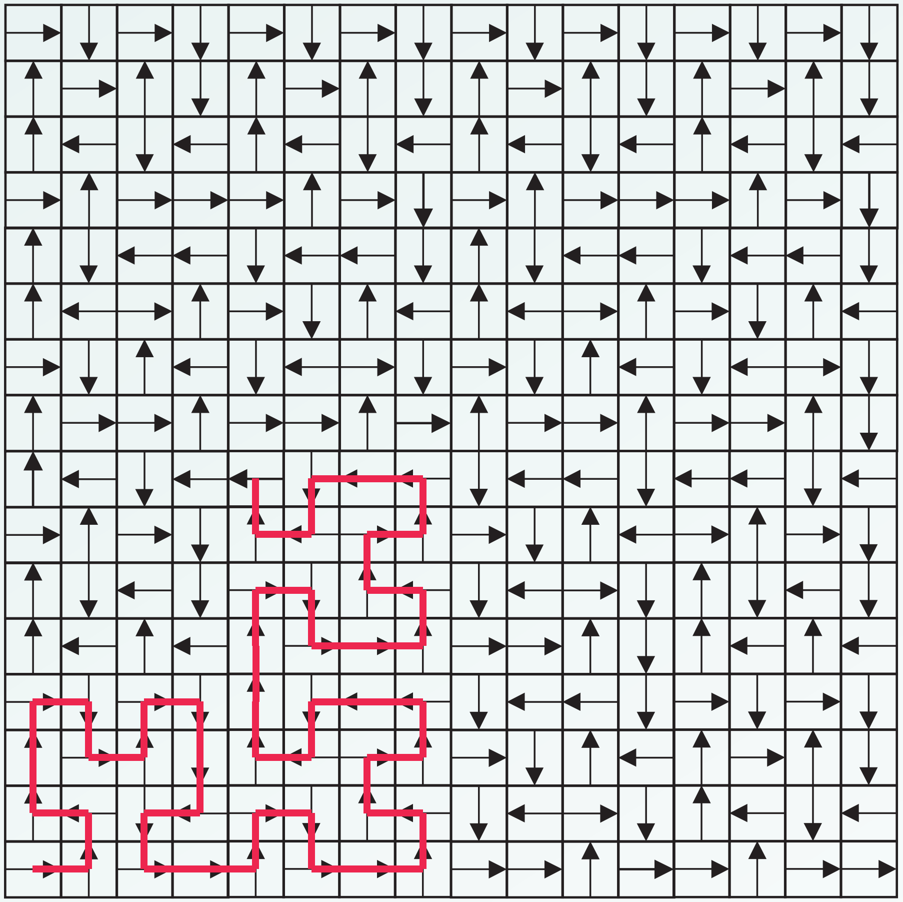

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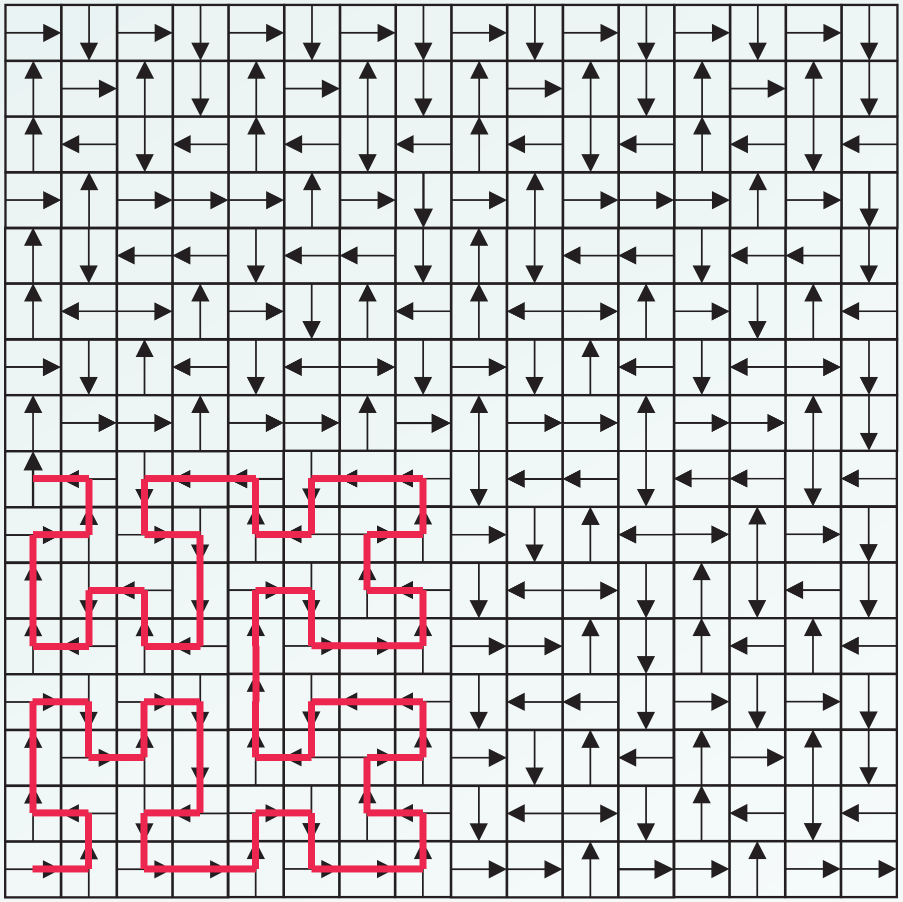

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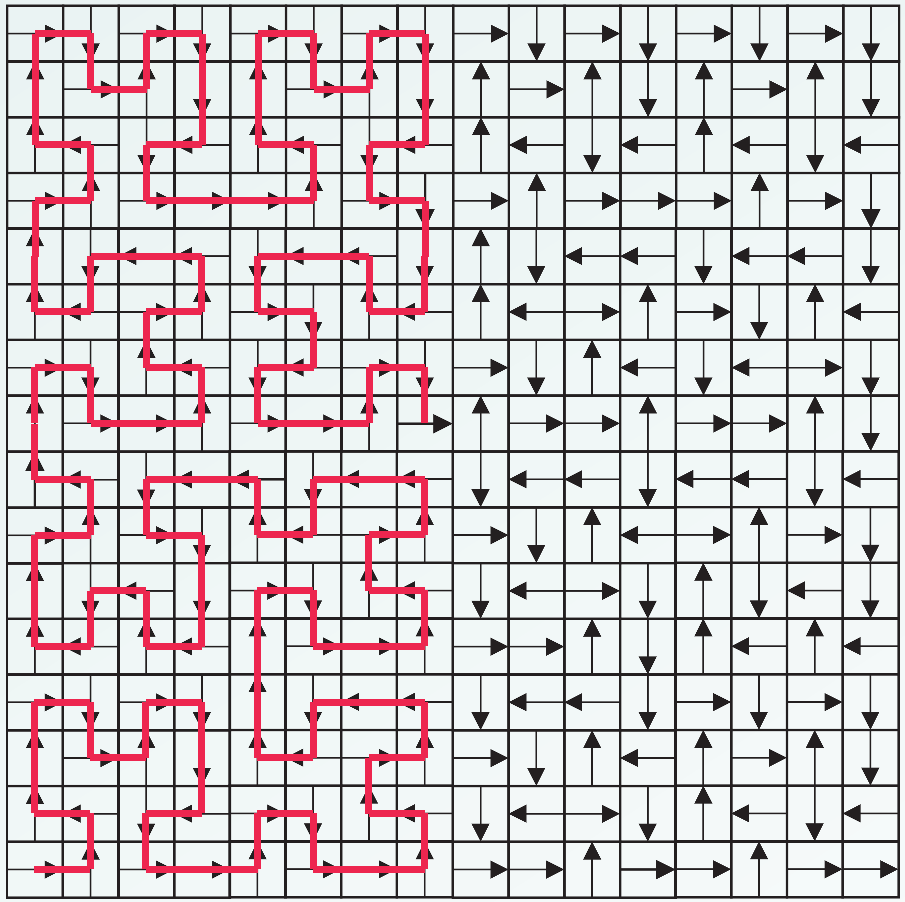

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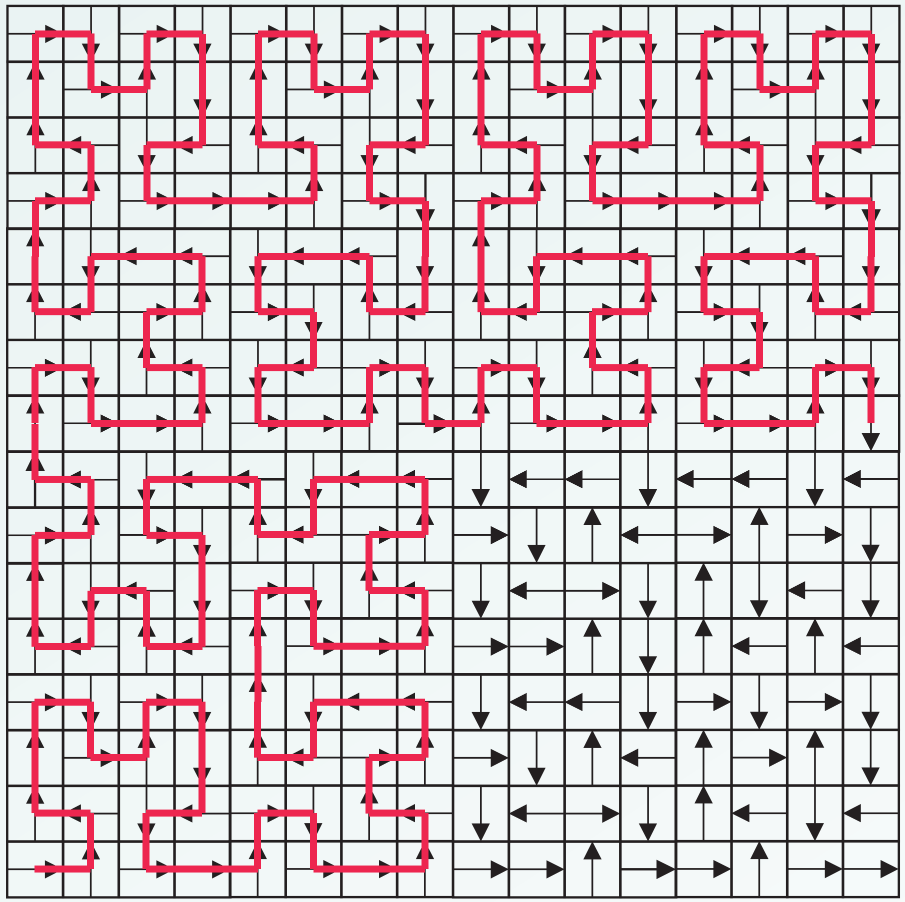

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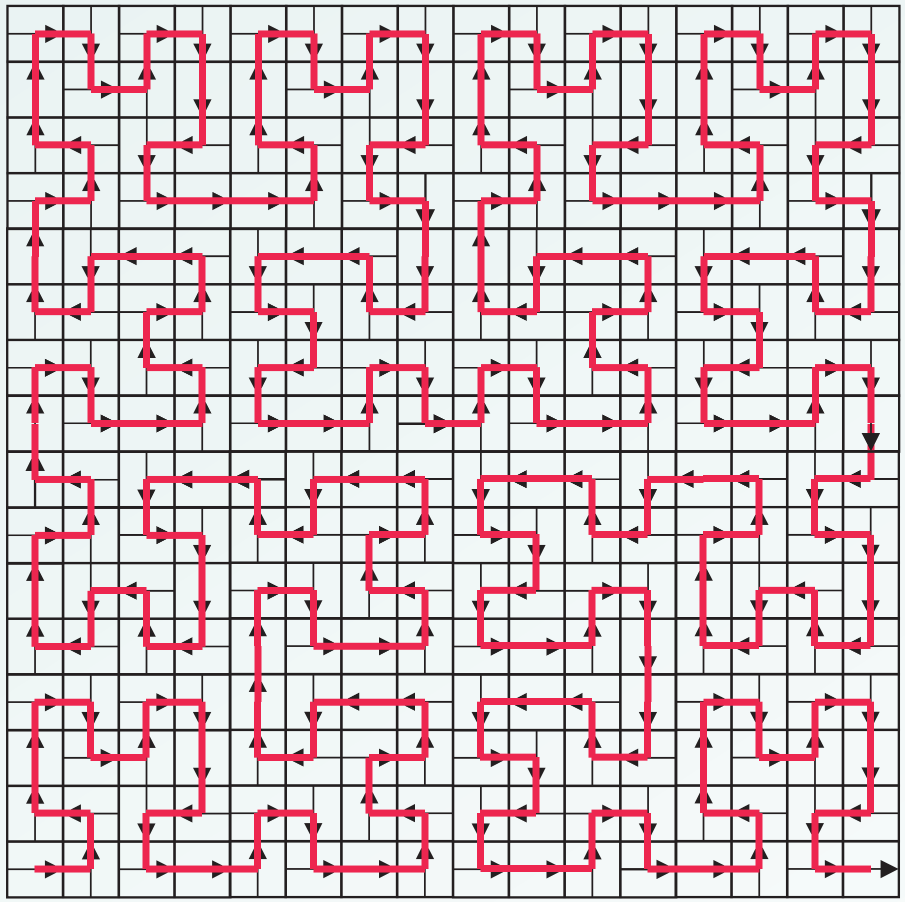


Applications of SNAKES

First application of SNAKES: An example of a two-dimensional CA that is injective on periodic configurations but is not injective on all configurations.

The **Snake XOR** CA confirms that in 2D

$$G \text{ injective} \quad \not\Leftarrow \quad G_P \text{ injective.}$$

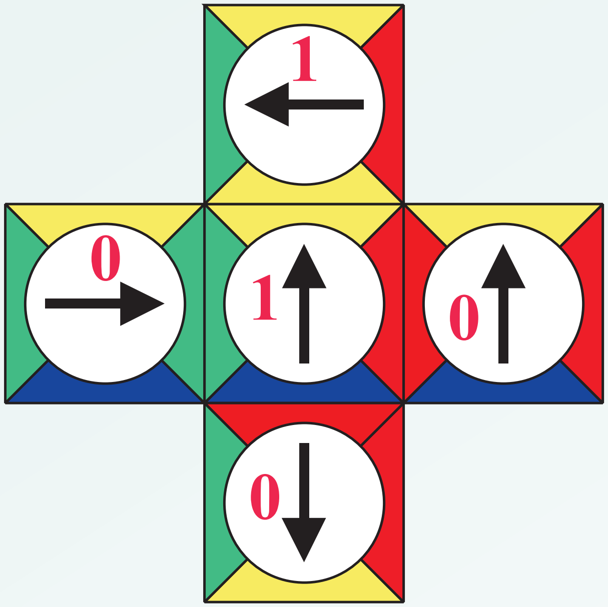
The state set of the CA is

$$S = \textsc{Snakes} \times \{0, 1\}.$$

(Each snake tile is attached a red bit.)

The local rule checks whether the tiling is valid at the cell:

- If there is a tiling error, no change in the state.



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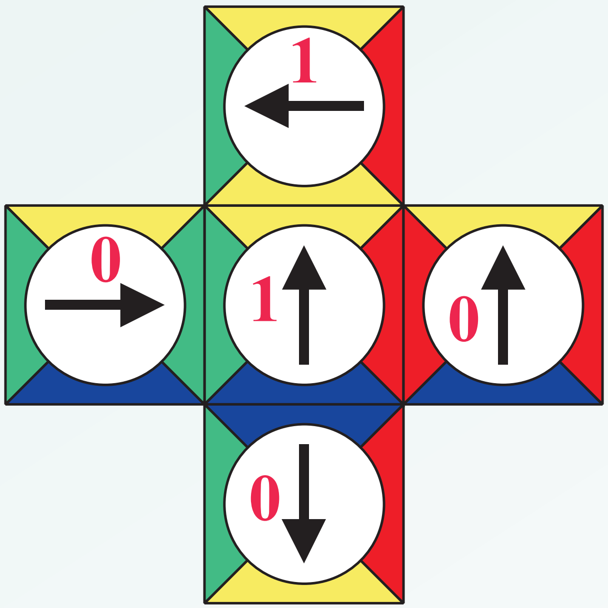
- If there is a tiling error, no change in the state.
- If the tiling is valid, the cell is **active**: the bit of the neighbor next on the path is XOR'ed to the bit of the cell.



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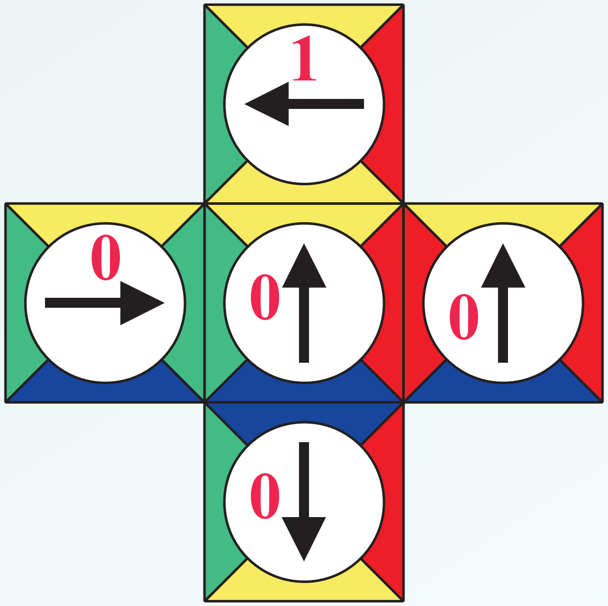

Snake XOR is not injective:

The following two configurations have the same successor: The SNAKES tilings of the configurations form the same valid tiling of the plane. In one of the configurations all bits are set to 0, and in the other configuration all bits are 1.

All cells are active because the tilings are correct. This means that all bits in both configurations become 0. So the two configurations become identical. The CA is not injective.

Snake XOR is injective on periodic configurations:

Suppose there are different periodic configurations c and d with the same successor. Since only bits may change, c and d must have identical SNAKES tiles everywhere. So they must have different bits 0 and 1 in some position $\vec{p}_1 \in \mathbb{Z}^2$.

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Because c and d have identical successors:

- The cell in position $\vec{p}_1$ must be active, that is, the SNAKES tiling is valid in position $\vec{p}_1$.
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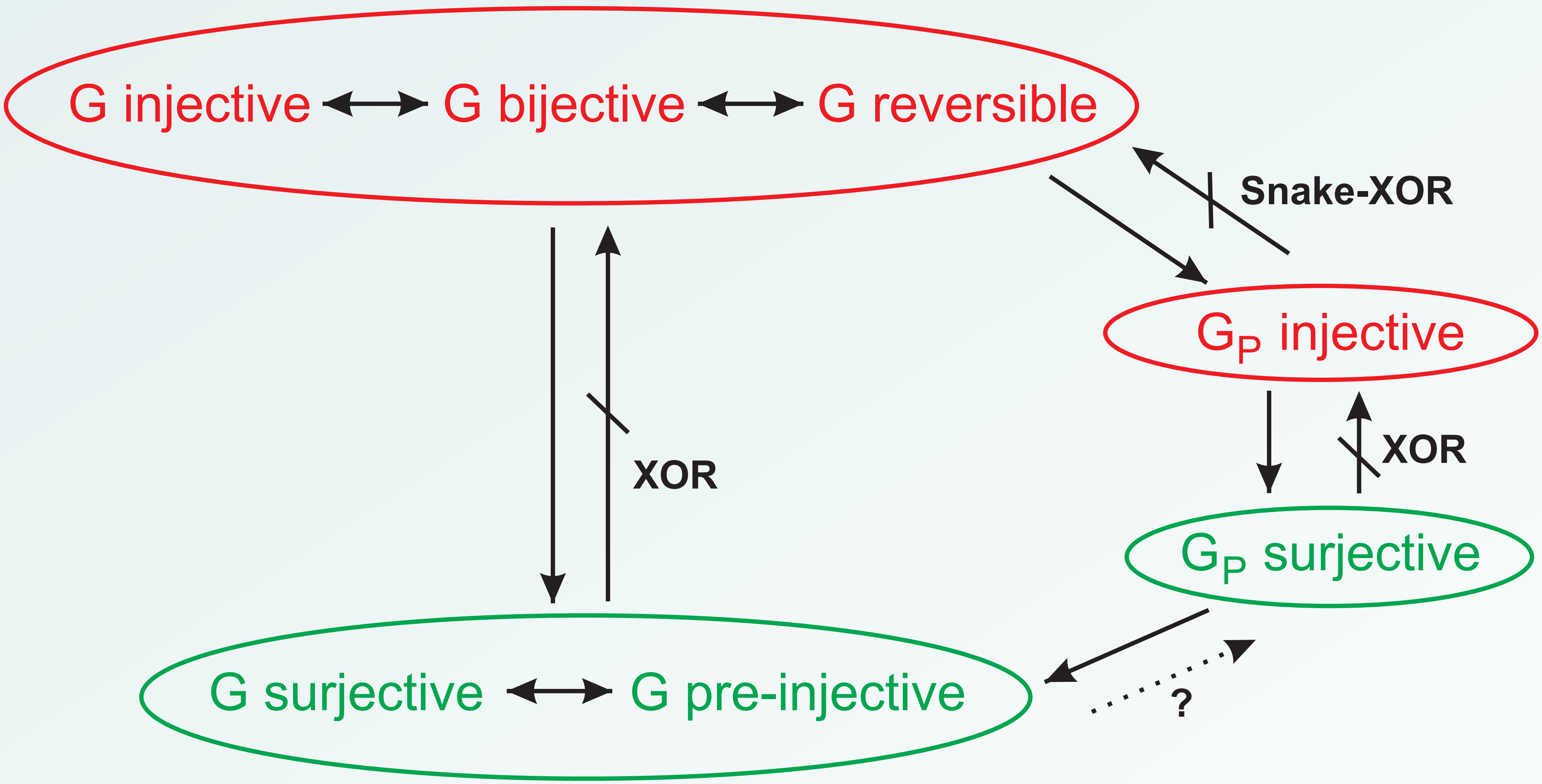
- The bits stored in the next position $\vec{p}_2$ (indicated by the direction) are different in c and d.

Hence we can repeat the reasoning in position $\vec{p}_2$.

The same reasoning can be repeated over and over again. The positions $\vec{p}_1, \vec{p}_2, \vec{p}_3, \ldots$ form a path that follows the arrows on the tiles. There is no tiling error at any tile on this path.

But this contradicts the fact that the plane filling property of SNAKES guarantees that on periodic configuration every path encounters a tiling error. $\square$

In 2D


G injective
G bijective
G reversible
Snake-XOR
G_P injective
XOR
XOR
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G surjective
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?

Second application of Snakes: It is undecidable to determine if a given two-dimensional CA is reversible.

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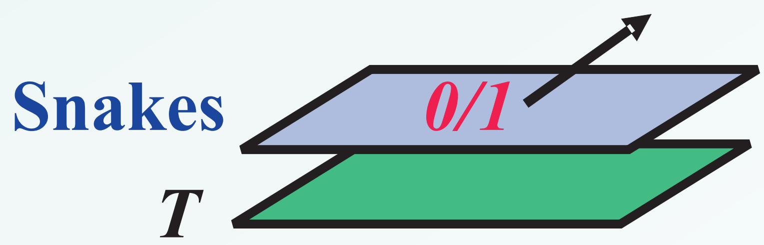
The proof is a reduction from the tiling problem, using the tile set SNAKES.

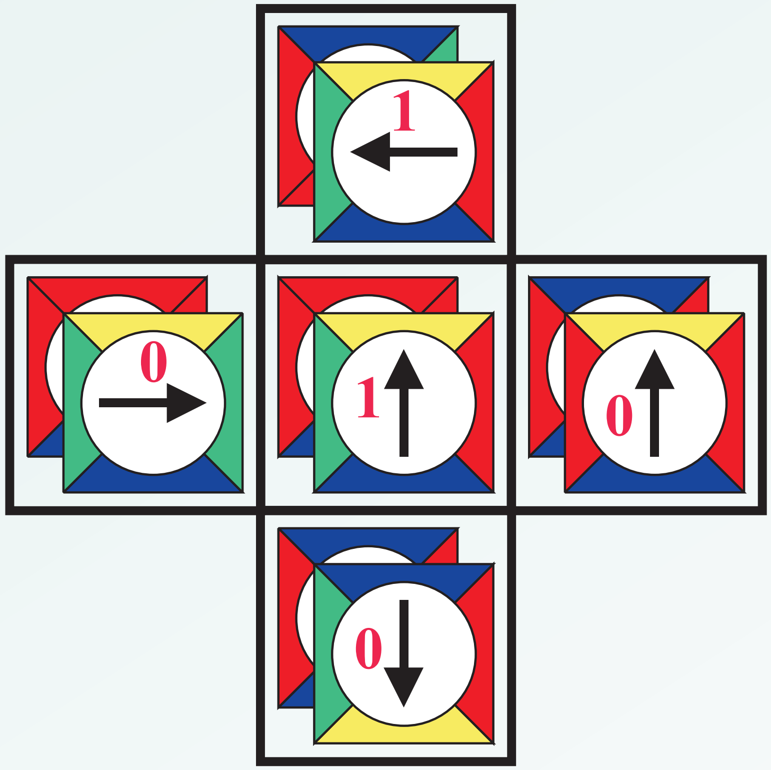
For any given tile set T we construct a CA with the state set

$$S = T \times \text{SNAKES} \times \{0, 1\}.$$



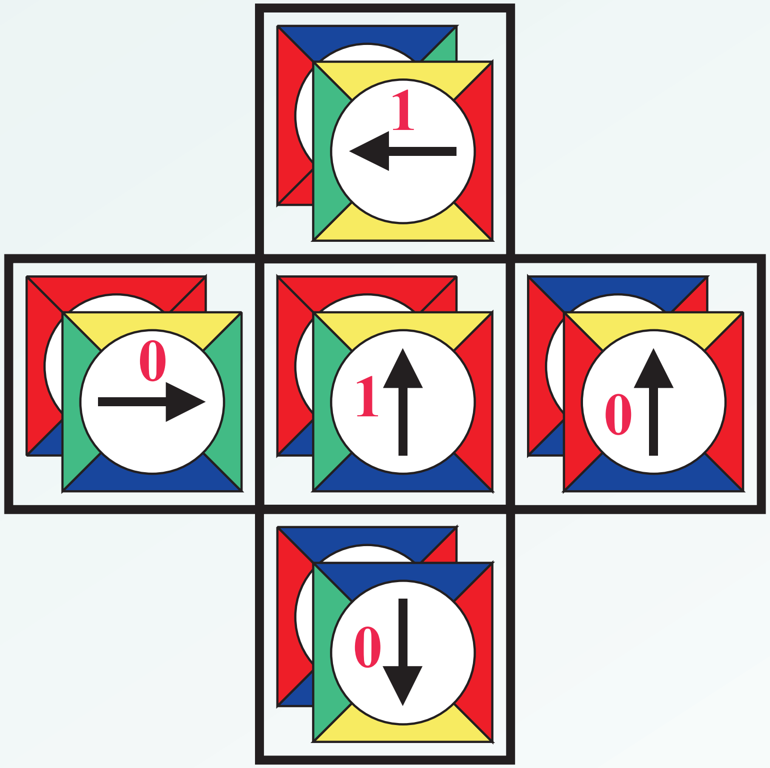
The local rule is analogous to **Snake XOR** with the difference that the correctness of the tiling is checked in both tile layers:

- If there is a tiling error then the cell is inactive.



The local rule is analogous to **Snake XOR** with the difference that the correctness of the tiling is checked in both tile layers:

- If there is a tiling error then the cell is inactive.
- If both tilings are valid, the bit of the neighbor next on the path is XOR'ed to the bit of the cell.



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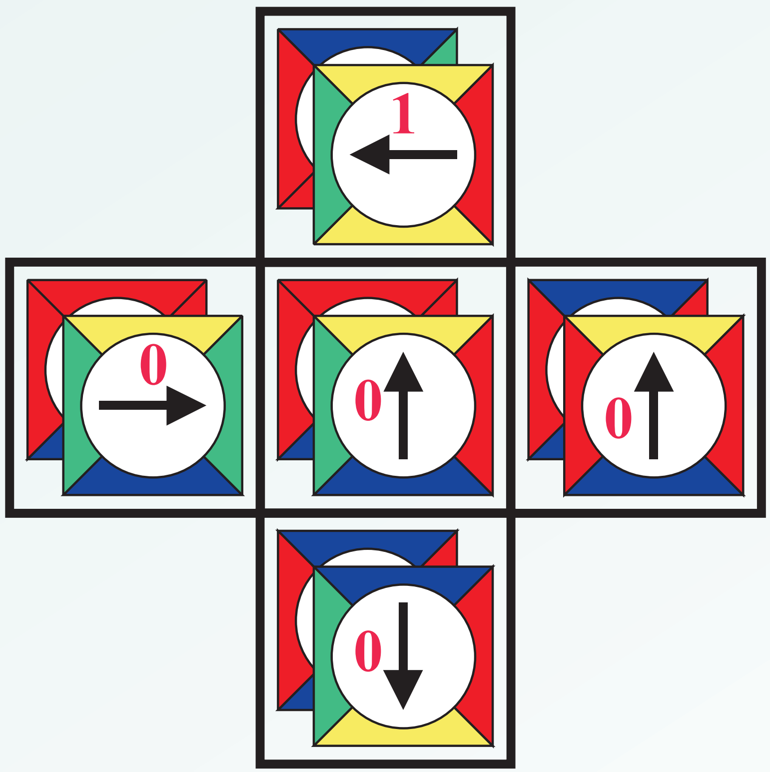

We can reason exactly as with **Snake XOR**, and show that the CA is reversible if and only if the tile set T does not admit a plane tiling:

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($\Longrightarrow$) If a valid tiling of the plane exists then we can construct two different configurations of the CA that have the same image under G. The SNAKES and the T layers of the configurations form the same valid tilings of the plane. In one of the configurations all bits are 0, and in the other configuration all bits are 1.

All cells are active because the tilings are correct. This means that all bits in both configurations become 0. So the two configurations become identical. The CA is not injective.

($\Longleftarrow$) Conversely, assume that the CA is not injective. Let c and d be two different configurations with the same successor. Since only bits may change, c and d must have identical SNAKES and T layers. So they must have different bits 0 and 1 in some position $\vec{p}_1 \in \mathbb{Z}^2$.

($\Longleftarrow$) Conversely, assume that the CA is not injective. Let c and d be two different configurations with the same successor. Since only bits may change, c and d must have identical SNAKES and T layers. So they must have different bits 0 and 1 in some position $\vec{p}_1 \in \mathbb{Z}^2$.

Because c and d have identical successors:

- The cell in position $\vec{p}_1$ must be active, that is, the SNAKES and T tilings are both valid in position $\vec{p}_1$.
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The same reasoning can be repeated over and over again. The positions $\vec{p}_1, \vec{p}_2, \vec{p}_3, \ldots$ form a path that follows the arrows on the tiles. There is no tiling error at any tile on this path so the special property of SNAKES forces the path to cover arbitrarily large squares.

Hence T admits tilings of arbitrarily large squares, and consequently a tiling of the infinite plane. $\square$

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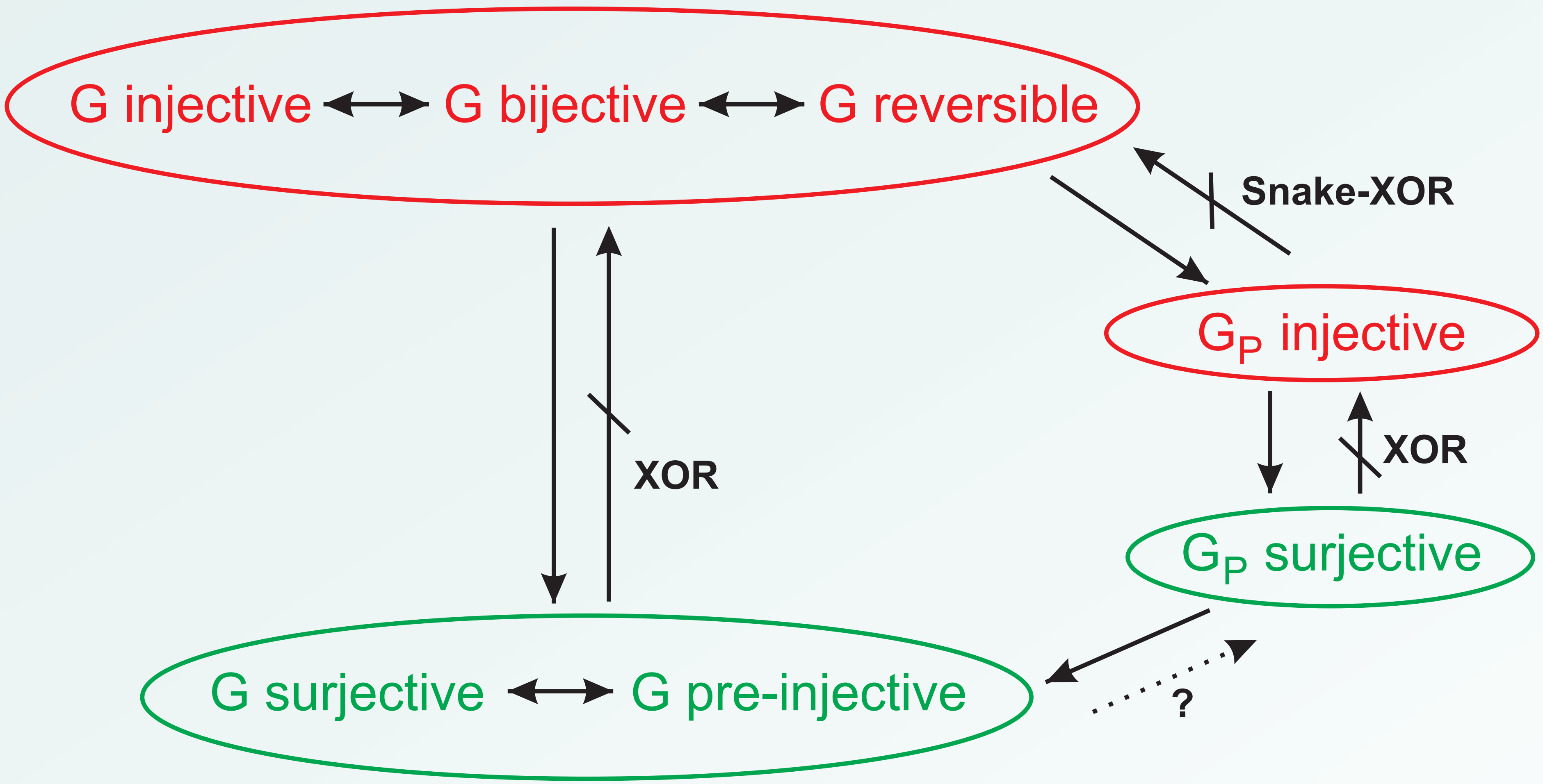
An analogous (but simpler!) construction can be made for the surjectivity problem, based on the fact surjectivity is equivalent to pre-injectivity:

Theorem: It is undecidable whether a given two-dimensional CA is surjective.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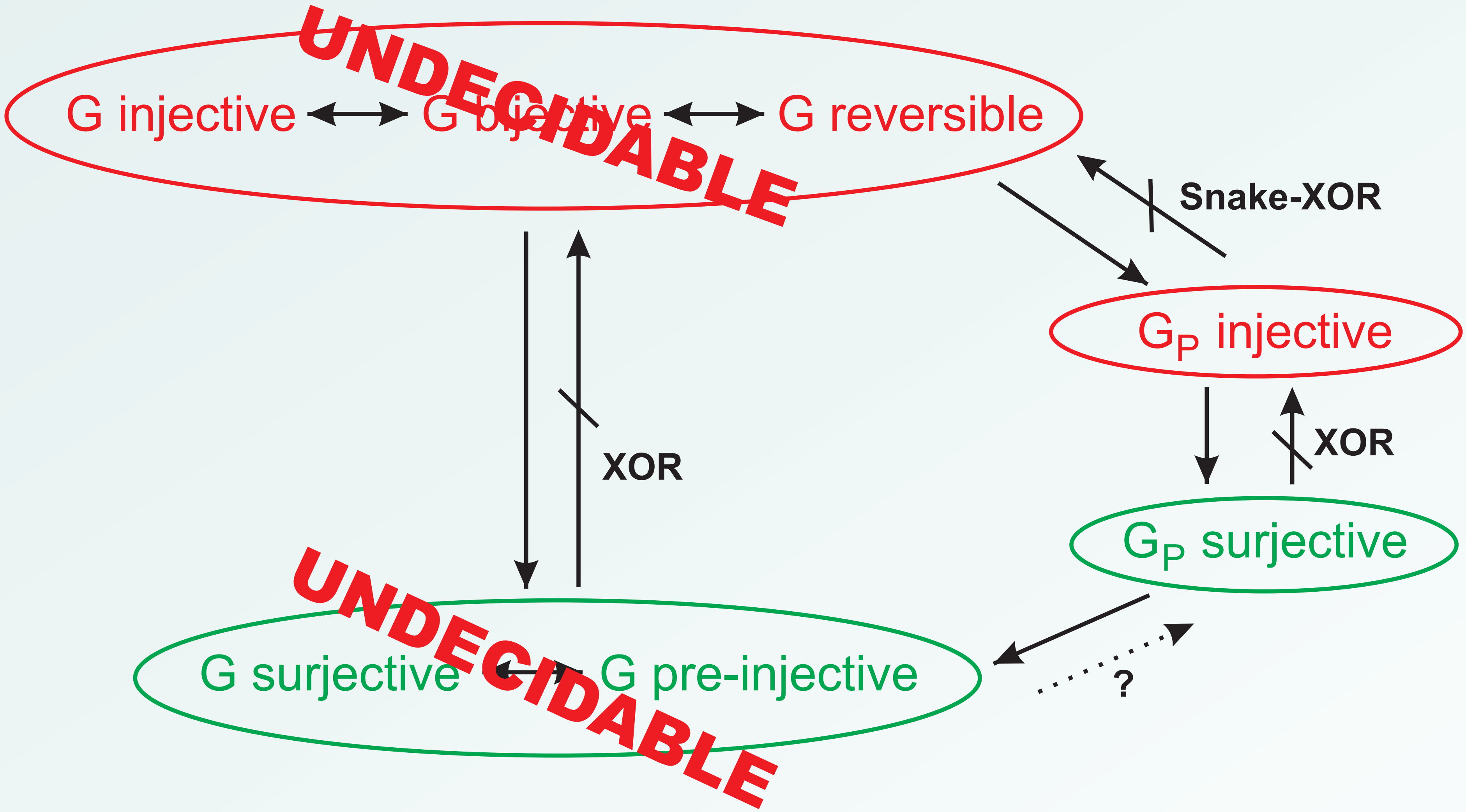

G injective
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UNDECIDABLE
Snake-XOR
XOR
G$_P$ injective
XOR
G$_P$ surjective
G surjective
G pre-injective
UNDECIDABLE
?

Both problems are semi-decidable in one direction:

Injectivity is semi-decidable: Enumerate all CA G one-by one and check if G is the inverse of the given CA. Halt once (if ever) the inverse is found.

Non-surjectivity is semi-decidable: Enumerate all finite patterns one-by-one and halt once (if ever) an orphan is found.

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There are some reversible CA that use von Neumann neighborhood but whose inverse automata use a very large neighborhood: There can be no computable upper bound on the extend of this inverse neighborhood.

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Topological arguments $\Longrightarrow$ A finite neighborhood is enough to determine the previous state of a cell.

Computation theory $\Longrightarrow$ This neighborhood may be extremely large.

Undecidability of surjectivity implies the following:

There are non-surjective CA whose smallest orphan is very large: There can be no computable upper bound on the extend of the smallest orphan.

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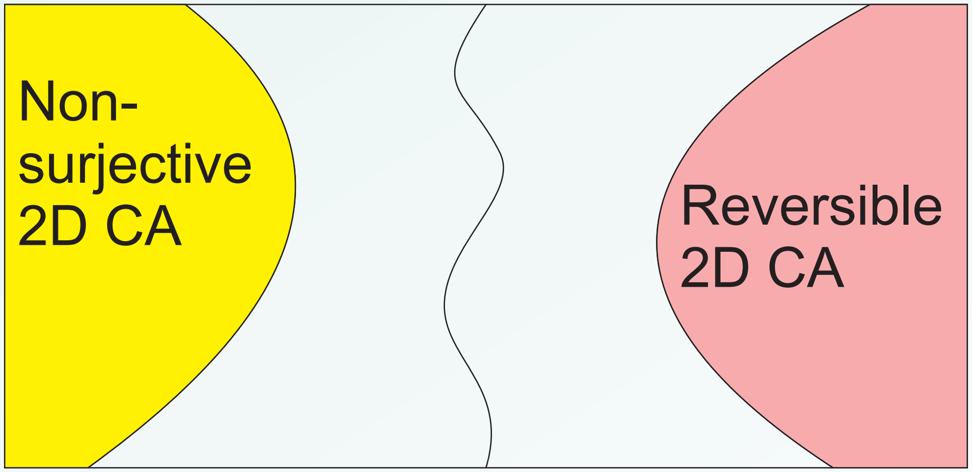
So while the smallest known orphan for Game-Of-Life is pretty big (92 cells), this pales in comparison with some other CA.

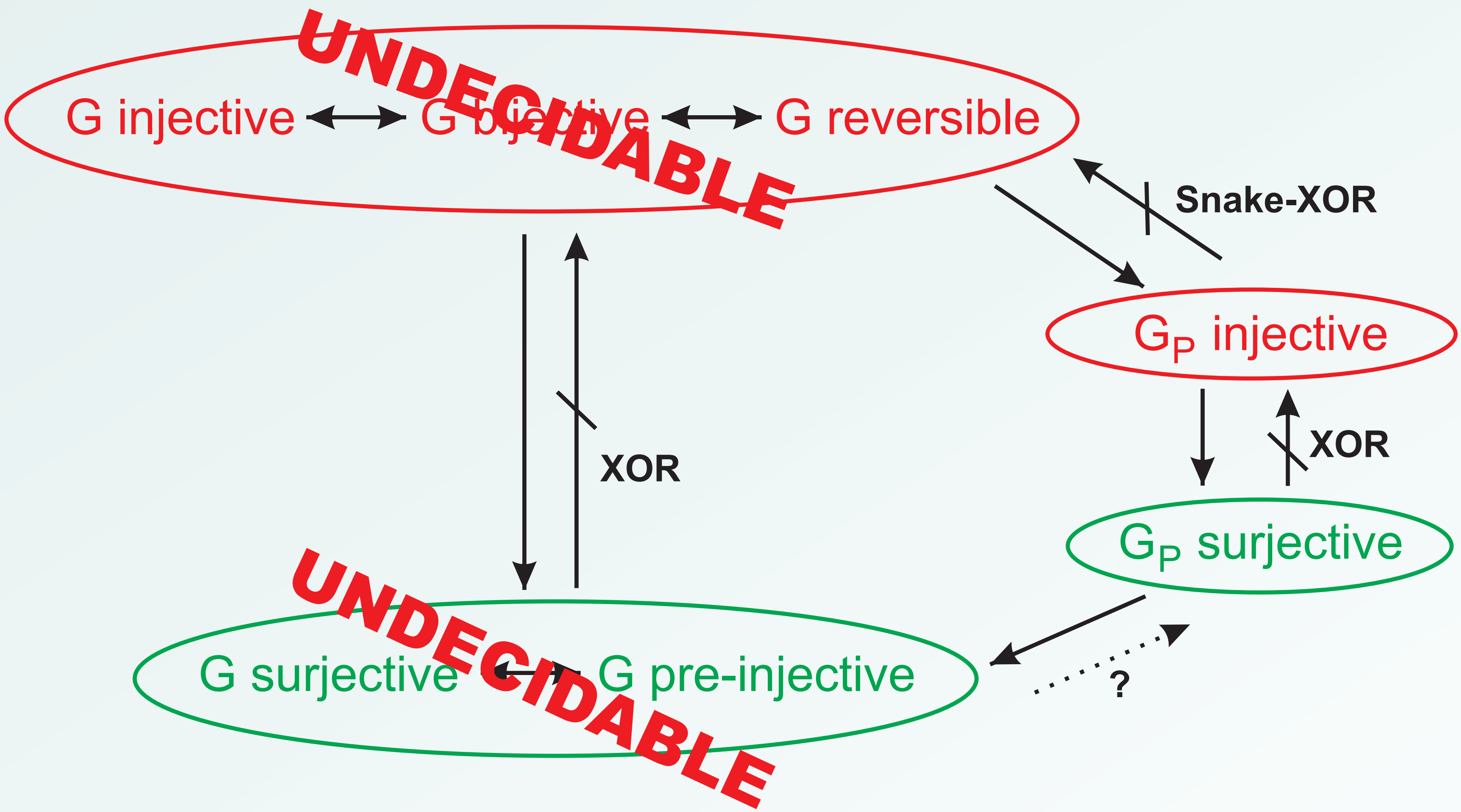
The undecidability proofs for reversibility and surjectivity can be merged into

Theorem: The classes of

- Reversible 2D CA
- Non-surjective 2D CA

are recursively inseparable





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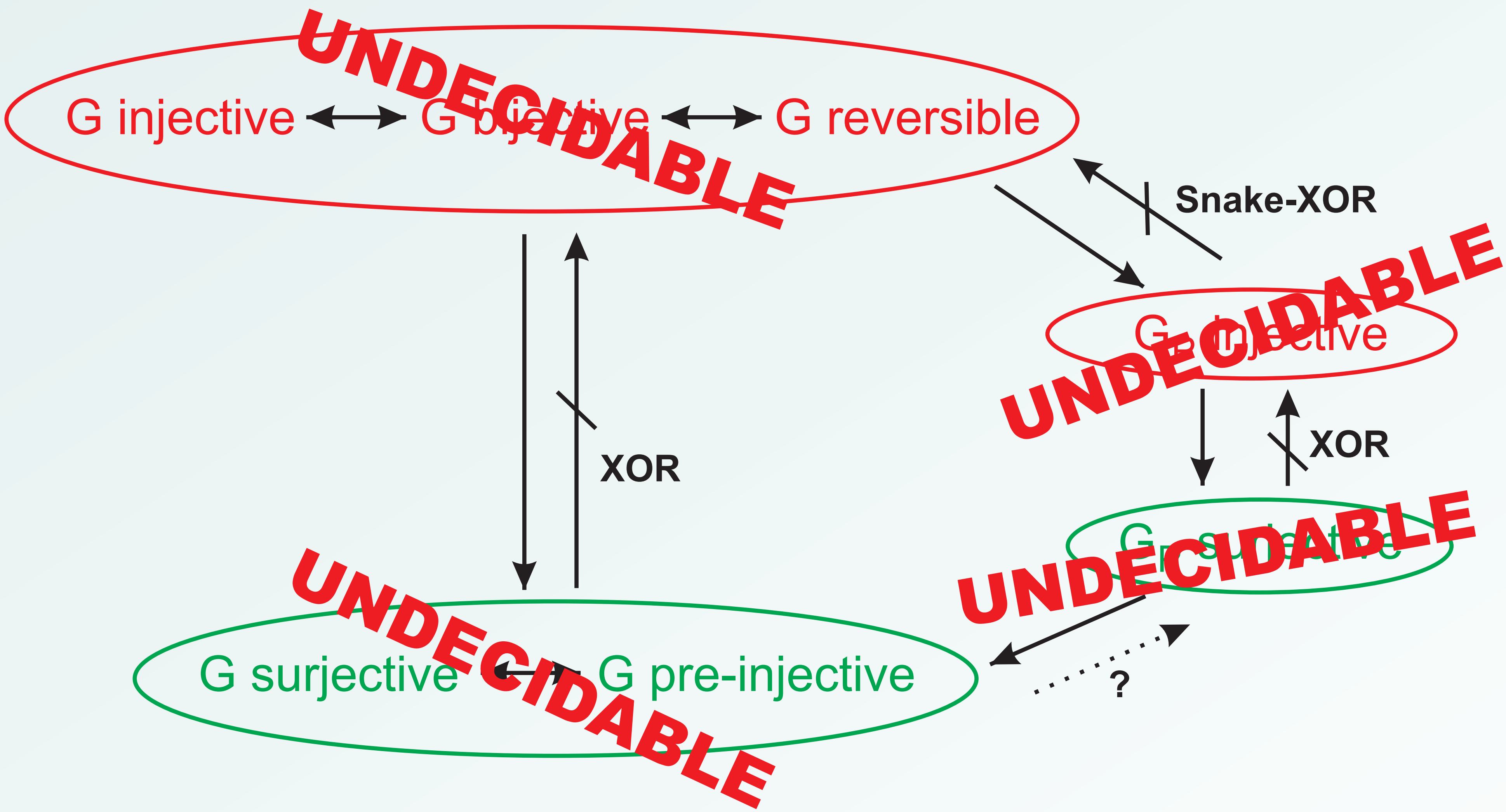

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Any solutions may be presented at CrapCon'15