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LOGICS OF ACTION

- happening vs doing vs action
- action as successful trying (agency, intentionality)
- action as exercised ability
- action $\neq$ act

an act is what ~~takes~~ takes place when an action is performed

- actions
  - executions (defined by what is executed)
  - accomplishments (defined by what is achieved)

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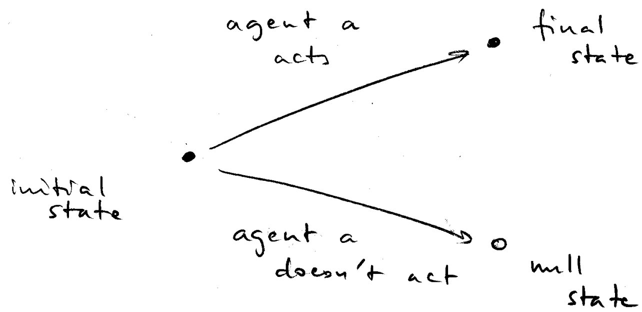
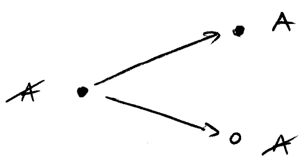
Two streams of logics

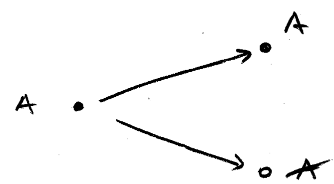
- logics of agency
  - von Wright, Fitch, Pörn, Brown; Chellas' Δ, Belnap and Perloff's STIT
- logics of actions
  - dynamic logic, Segerberg's δ

Von Wright's intuition

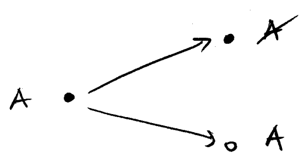




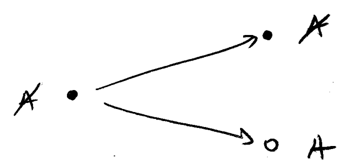
a produces A
(brings it about that)



a sustains A



a destroys A (= produces ¬A)



a suppresses A (= sustains ¬A)

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Chellas' logic of seeing-to-it-that (Δ-logic)

- seeing-to-it-that is done
  by exercising control over the present (narrowing
  the set of yet possible histories
  at the time immediately preceding
  the current time)
- avoidability is not required

Syntax of Δ-logic

Assumed are a denumerable set Fma_0 of prop. letters and a non-empty set Agt of agent names.

The set Fma of formulae is defined as follows:

- if $p \in Fma_0$, then $p \in Fma$;
- $\top, \bot \in Fma$;
- if $A \in Fma$, then $\neg A \in Fma$;
- ... ;
- if $a \in Agt$, $A \in Fma$, then $\Delta_a A$ (a sees to it that A) $\in Fma$;
- if $A \in Fma$, then $\boxdot A$ (it is historically necessary that A) $\in Fma$.

Δ- structures

A Δ-structure is a 6-tuple $\langle T, \leq, S, H, R, V \rangle$ where

- T is a (non-empty) set of something called <u>times</u> or <u>time instants</u>;
- $\leq \; \in \mathcal{P}(T \times T)$ is a binary relation on times which is <u>reflexive</u>, <u>antisymmetric</u>, <u>transitive</u> and <u>linear</u>, i.e. a <u>linear ordering</u> of T; if $t \leq t'$, say that t is <u>not later</u> than t';
- S is a (non-empty) set of something called <u>states-of-affairs</u> (<u>events</u>);
- $H \in \mathcal{P}([T \to S])$ is a set of functions from T to S; elements of H are called <u>(possible) histories</u>;

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define a function $= \in [T \to \mathcal{P}(H \times H)]$ from times to binary relations on histories by letting

* $h =_t h'$ iff, for any $t' < t$, $h_{t'} = h'_{t'}$;

If $h =_t h'$ say that h, h' have the <u>same past</u> at t;

H is required to meet the <u>future branching only</u> condition:

* for any $t \in T$, $h, h' \in H$,
if $h_t = h'_t$, then $h =_t h'$;

- $R \in [Agt \times T \to \mathcal{P}(H \times H)]$ is a function from agent names and times to binary relations on histories called the <u>instigative alternativeness relations</u>; for any $a \in Agt$, $t \in T$, the relation $R_{a,t}$ is required to be <u>reflexive</u> and R must meet the <u>historical relevance</u> condition:

  * for any $a \in Agt$, $t \in T$, $h, h' \in H$, if $h R_{a,t} h'$, then $h =_t h'$.

- $V \in [Fma_0 \to \mathcal{P}(S)]$ is a function from prop. letters to sets of states-of-affairs called the <u>valuation</u>.

Intuition

- $h \; R_{a,t} \; h'$

  $\Leftrightarrow$ history h' is an instigative alternative to history h for agent a at time t

  $\Leftrightarrow$ agent a at t would not be able to prevent h' from appearing to be the actual history, if h were the actual history

Satisfaction in Δ-structures

Given a Δ-structure $M = \langle T, \leq, S, H, R, V \rangle$, define the corresponding interpretation function $[\![-]\!]^M \in [Fma \to \mathcal{P}(T \times H)]$ from formulae to time-history pairs as follows:

* If $p \in Fma_0$, then: $\models^M_{t,h} p$ iff $h_t \in V(p)$;
* $\models^M_{t,h} \top$ is true;
* $\models^M_{t,h} \bot$ is false;
* $\models^M_{t,h} \neg A$ iff not $\models^M_{t,h} A$;
* $\ldots$;
* $\models^M_{t,h} \Delta_a A$ iff, for any $h' \in H$ st $h R_{a,t} h'$, $\models^M_{t,h'} A$;
* $\models^M_{t,h} \boxdot A$ iff, for any $h' \in H$ st $h =_t h'$, $\models^M_{t,h'} A$.

A formula A is valid in M (notation $\models^M A$), if $\models^M_{t,h} A$, for any $t \in T$, $h \in H$.

Intuition

- $\Delta_a A$ holds at time t in history h

$\iff$ A holds at time t in any history h' that is an instigative alternative to h for agent a at t

$\iff$ A would hold at t in any history h' that agent a at t would not be able to prevent from appearing to be the actual history, if the actual history were h

Some observations

- The following hold in any Δ-structure M:

  * $\models^M \Delta_a(A \supset B) \supset (\Delta_a A \supset \Delta_a B)$;
    If $\models^M A$, then $\models^M \Delta_a A$; $\models^M \Delta_a \top$;

  * $\models^M \Delta_a A \supset A$
    (reliability);

  * $\models^M \Delta_a \Delta_b A \supset \Delta_a A$
    (qui facit per alium, facit per se; if a sees to it that b sees to it that something, then a sees to it, too);

  * $\models^M \boxdot A \supset \Delta_a A$;

  * $\models^M \boxdot A \supset A$;
    $\models^M \boxdot A \supset \boxdot\boxdot A$;
    $\models^M \neg\boxdot A \supset \boxdot\neg\boxdot A$.

Belnap and Perloff's logic of seeing-to-it-that
(stit-logic)

- seeing-to-it-that results from a prior choice
  (the choices at any time are given by a partition of the histories yet possible at that moment)
- avoidability is required

Syntax of stit-logic

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Assumed are again a denumerable set Fma_0 of prop. letters and a non-empty set Agt of agent names.

The set Fma of formulae is defined as follows:

- If $p \in Fma_0$, then $p \in Fma$;
- $\top, \bot \in Fma$;
- If $A \in Fma$, then $\neg A \in Fma$;
- ...;
- If $a \in Agt$, $A \in Fma$, then $stit_a A$ (a sees to it that A) $\in Fma$.

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stit-structures

A stit-structure is a 6-tuple $\langle T, \leq, S, H, E, V \rangle$ where

- T is a (non-empty) set of something called <u>times</u> or <u>time instants</u>;
- $\leq \in \mathcal{P}(T \times T)$ is a <u>linear ordering</u> of T, the <u>no-later-than</u> relation;
- W is a (non-empty) set of something called <u>states-of-affairs</u>
- $H \in \mathcal{P}([T \to S])$ is a set of functions from times to worlds; elements of H are called <u>(possible) histories</u>;

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define a function $\equiv \in [T \to P(H \times H)]$ from times to binary relations on histories by letting

* $h \equiv_t h'$ iff, for any $t' \leq t$, $h_{t'} = h'_{t'}$;

if $h \equiv_t h'$ say that h, h' have the <u>same past and present</u> at t;

H is required to meet the <u>future branching only</u> condition;

* for any $t \in T$, $h, h' \in H$,
  if $h_t = h'_t$, then $h \equiv_t h'$;

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- $E \in [Agt \times T \to P(H \times H)]$ is a function from agent names and times to binary relations on histories called the <u>same choice</u> relations;

for any $a \in Agt$, $t \in T$, the relation $E_{a,t}$ is required to be <u>reflexive</u>, <u>symmetric</u> and <u>transitive</u> (ie, an <u>equivalence</u>) and to meet the following conditions:

* for any $a \in Agt$, $t \in T$, $h, h' \in H$,
  if $h\, E_{a,t}\, h'$, then $h \equiv_t h'$
  (<u>historical relevance</u>);
* for any $a \in Agt$, $t \in T$, $t' > t$, $h, h' \in H$,
  if $h \equiv_{t'} h'$, then $h\, E_{a,t}\, h'$
  (<u>no choice between undivided histories</u>);
* for any $t \in T$, $\{h_a\}_{a \in Agt} \subseteq H$,
  there exists a $h \in H$, st $h_a\, E_{a,t}\, h'$
  for all $a \in Agt$ (<u>something happens</u>).

(102)

- $V \in [Fma_0 \to P(S)]$ is a function from prop. letters to sets of states-of-affairs called the <u>valuation</u>.

(103)

<u>Intuition</u>

- $h \; E_{a,t} \; h'$

  $\iff$ if h were the actual history and agent a made a choice at t, then, by this choice, he would not be able to prevent h' from appearing to be the actual history

Satisfaction in stit-structures

Given a stit-structure $M = \langle T, \leq, S, H, E, V \rangle$, define the interp. function $[\![-]\!]^M \in [\mathrm{Fm}_a \to \mathcal{P}(T \times H)]$ from formulae to time-history pairs as follows:

* if $p \in \mathrm{Fm}_{a_0}$, then: $\vDash^M_{t,h} p$ iff $h_t \in V(p)$;
* $\vDash^M_{t,h} \top$ is true;
* $\vDash^M_{t,h} \bot$ is false;
* $\vDash^M_{t,h} \neg A$ iff not $\vDash^M_{t,h} A$;
* ... ;
* $\vDash^M_{t,h}$ stit$_a$ A iff, for some $t' < t$ (a witnessing choice point)
  (1) for any $h' \in H$ st $h \, E_{a,t'} \, h'$, $\vDash^M_{t,h'} A$ (the positive condition);
  (2) for some $h' \in H$ st $h \equiv_{t'} h'$, not $\vDash^M_{t,h'} A$ (the negative condition).

(105)

Intuition

- $stit_a\, A$ holds at time t in history h

$\iff$ there is a prior time t' st,
if h were the actual history and
agent a had made a choice at time t',
then

(1) A would hold at t in any history h' that a, by his choice at t', would not have been able to prevent from becoming actual;

(2) A would not hold at t in some history h' that, at t', would have had the same past and present as h.

(106)

Observations

For any stit-structure M, the following are true:

* if $\models^M A \supset B$ and t_1 witnesses $\models^M_{t,h} stit_a\, A$, t_2 witnesses $\models^M_{t,h} stit_a\, B$, then $t_1 \geq t_2$; (1)

* if $\models^M A \equiv B$ and t_1 witnesses $\models^M_{t,h} stit_a\, A$, t_2 witnesses $\models^M_{t,h} stit_a\, B$, then $t_1 = t_2$;

* if $\models^M_{t,h} stit_a\, A$, then there is exactly one witness for this.

Verification of (1):

Suppose that, to the contrary, we have $A, B \in Fma$, $t, t_1, t_2 \in T$, $h \in H$ st $\models^M A \supset B$, t_1 witnesses $\models^M_{t,h}$ stit$_a$ A, t_2 witnesses $\models^M_{t,h}$ stit$_a$ B and $t_2 > t_1$. Let us see that this leads to a contradiction.

From the <u>negative</u> condition for t_2 witnessing $\models^M_{t,h}$ stit$_a$ B, it follows that there is a $h \in H$ st $h \equiv_{t_2} h'$ and not $\models^M_{t,h'}$ B. Since $\models^M A \supset B$, it must also be the case that not $\models^M_{t,h'}$ A. From M meeting the <u>no choice between undivided histories</u> condition, and $t_2 > t_1$, it follows that $h \; E_{a,t_1} \; h'$. This, together with the <u>positive</u> condition for t_1 witnessing $\models^M_{t,h}$ stit$_a$ A, yields $\models^M_{t,h'}$ A. Contradiction.

<u>Observations</u>

For any stit-structure M, the following are true:

* if $\models^M A \equiv B$, then $\models^M$ stit$_a$ $A \equiv$ stit$_a$ B;
* $\models^M$ stit$_a$ $A \supset A$;
* $\models^M \neg$ stit$_a$ $\top$;
* $\models^M$ stit$_a$ $A \wedge$ stit$_a$ $B \supset$ stit$_a$ $(A \wedge B)$;
* $\models^M$ stit$_a$ $A \supset$ stit$_a$ stit$_a$ A;
* $\models^M$ stit$_a$ stit$_b$ $A \supset$ stit$_a$ A;
* $\models^M \neg$ stit$_a$ stit$_b$ A (whenever $a \neq b$).

Segerberg's logic of bringing it about

(109)

(δ-logic)

- essentially a version of dynamic logic
- δA - a name for the action of bringing it about that A

(cf in Chellas' logic,

ΔA - a name for the proposition that A is, or has been, brought about)

(110)

Syntax of δ-logic

Assumed is a denumerable set $Fma_0 = \{p_0, p_1, \dots\}$ of prop. letters.

The set Fma of formulae and the set Prg of programs are defined as follows:

* if $P, Q \in Prg$, then $P;Q$, $P \cup Q \in Prg$;
* if $P \in Prg$, then $P^* \in Prg$;
* if $A \in Fma$, then δA (bringing it about that A) $\in Prg$
* if $p \in Fma_0$, then $p \in Fma$;
* $\top, \bot \in Fma$;
* if $A \in Fma$, then $\neg A \in Fma$;
* ...
* if $P \in Prg$, $A \in Fma$, then $[P]A$, $\langle P \rangle A \in Fma$.

δ-structures

A δ-structure is a triple $M = (W, D, V)$ where

* W is a non-empty set of something called <u>states</u>;
* $D \in [\mathcal{P}(W) \to \mathcal{P}(W,W)]$ is a function from sets of states to binary relations on states;
* D is required to meet two conditions:
  + for any $X \subseteq W$, $w \in W$,
    for any $w' \in W$ st $w D(X) w'$, $w' \in X$
    (<u>reliability</u>)
  + for any $X, Y \in W$, $w \in W$,
    if, for any $w' \in W$ st $w D(X) w'$, $w' \in Y$,
    then, for any $w' \in W$ st $w D(X) w'$, $w D(Y) w'$
    (<u>weak maximality</u>)
* $V \in [Fm_{A_0} \to \mathcal{P}(W)]$ is a function from prop. letters to sets of states.

Satisfaction in δ-structures

(112)

Given a δ-structure $M = (W, D, V)$, the corresponding interpretation functions $\{\!|-|\!\}^M \in [Prg \to \mathcal{P}(W \times W)]$ from programs to binary relations on states and $[\![-]\!]^M \in [Fma \to \mathcal{P}(W)]$ from formulae to sets of states are defined as follows ($\models^M_w A$ is short for $w \in [\![A]\!]^M$):

* $w \{\!|P;Q|\!\}^M w'$ iff, for some $w'' \in W$, $w \{\!|P|\!\}^M w''$ and $w'' \{\!|Q|\!\}^M w'$;
* $w \{\!|P \cup Q|\!\}^M w'$ iff $w \{\!|P|\!\}^M w'$ or $w \{\!|Q|\!\}^M w'$;
* $w \{\!|P^*|\!\}^M w'$ iff, for some $n \geq 0$, $w_0, \ldots, w_n \in W$, $w = w_0$, $w_0 \{\!|P|\!\}^M w_1, \ldots, w_{n-1} \{\!|P|\!\}^M w_n$, $w_n = w'$;
* $w \{\!|\delta A|\!\}^M w'$ iff $w \, D([\![A]\!]^M) \, w'$;
* if $p \in Fma_0$, then: $\models^M_w p$ iff $w \in V(p)$;
* $\models^M_w \top$ is true;
* $\models^M_w A \wedge B$ iff $\models^M_w A$ and $\models^M_w B$;
* $\ldots$;
* $\models^M_w [P] A$ iff, for any $w' \in W$ st $w \{\!|P|\!\}^M w'$, $\models^M_{w'} A$;
* $\models^M_w \langle P \rangle A$ iff, for some $w' \in W$ st $w \{\!|P|\!\}^M w'$, $\models^M_{w'} A$.

Observations

(113)

For any δ-structure M, the following hold:

* $\models^M [P](A \supset B) \supset ([P]A \supset [P]B)$;
* if $\models^M A$, then $\models^M [P]A$;
* $\models^M [\delta A] A$; (1)
* $\models^M [\delta A] B \supset ([\delta B] C \supset [\delta A] C)$; (2)
* if $\models^M A \supset B$, then $\models^M [P]A \supset [P]B$;
* if $\models^M A \supset B$, then $\models^M [\delta B] C \supset [\delta A] C$.

Verification for (1):

Consider any $w \in W$. We have to show that $\models^M_w [\delta A] A$, ie, that, for any $w' \in W$ st $w\, D([\![A]\!]^M)\, w'$, $\models^M_{w'} A$. But this is guaranteed by the <u>reliability condition</u> which is met by M.

Verification of (2):

Consider any $w \in W$. Assume that $\models^M_w [\delta A] B$, ie, that, for any $w' \in W$, st $w\, D([\![A]\!]^M)\, w'$, $\models^M_{w'} B$. Since M meets the <u>weak maximality condition</u>, this entails that, for any $w' \in W$ st $w\, D([\![A]\!]^M)\, w'$, $w\, D([\![B]\!]^M)\, w'$. Assume now also that $\models^M_w [\delta B]\, C$. We have to show that $\models^M_w [\delta A]\, C$, ie, that, for any $w' \in W$ st $w\, D([\![A]\!]^M)\, w'$, $\models^M_{w'} C$. Consider any such $w' \in W$ st $w\, D([\![A]\!]^M)\, w'$. We know that $w\, D([\![B]\!]^M)\, w'$. Since $\models^M_w [\delta B] C$, from this it follows that $\models^M_{w'} C$, and this was what we wanted to prove.

Axiomatization

δ-logic is (soundly & completely) axiomatized by $\mathbb{K}_{Prg}$ plus

$$[P;Q]A \equiv [P][Q]A$$

$$[P \cup Q]A \equiv [P]A \wedge [Q]A$$

$$[P^*]A \supset A \wedge [P][P^*]A$$

$$\frac{B \supset A \wedge [P]B}{B \supset [P^*]A}$$

$$[\delta A]A$$

$$[\delta A]B \supset ([\delta B]C \supset [\delta A]C)$$