

PREDIKAATLOOGIKA: LOOMULIK TULETUS

- Standardesitus: lauseloogika loomuliku tuletuse standardesituse reeglitele lisanduvad järgmised:

$$\frac{A[y/x]}{\forall x A} \quad \forall \mathcal{I}^*$$

$$\frac{\forall x A}{A[t/x]} \quad \forall \mathcal{E}$$

$$\frac{A[t/x]}{\exists x A} \quad \exists \mathcal{I}$$

$$\frac{\exists x A \quad \begin{array}{c} A[y/x] \\ \vdots \\ C \end{array}}{C} \quad \exists \mathcal{E}^\dagger$$

* y ei tohi vabalt esineda valemis $\forall x A$ ja kasutada olevates hüpoteesides

† y ei tohi vabalt esineda valemites $\exists x A, C$ ja kasutada olevates hüpoteesides

- Näiteid tõestustest:

$$\begin{array}{c}
 \begin{array}{c}
 +2 \\
 \frac{\forall y p(x', y)}{p(x', y')} \quad \forall\mathcal{E} \\
 +1 \\
 \frac{\exists x \forall y p(x, y) \quad \frac{p(x', y')}{\exists x p(x, y')} \quad \exists\mathcal{I}}{\exists x p(x, y')} \quad \exists\mathcal{E}, -2, x' \text{ värske} \\
 \frac{\exists x p(x, y')}{\forall y \exists x p(x, y)} \quad \forall\mathcal{I}, y' \text{ värske} \\
 \frac{\forall y \exists x p(x, y)}{\exists x \forall y p(x, y) \supset \forall y \exists x p(x, y)} \quad \supset \mathcal{I}, -1
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 +2 \\
 \frac{\forall x \neg p(x)}{\neg p(x')} \quad \forall\mathcal{E} \\
 +1 \\
 \frac{\exists x p(x) \quad \frac{\neg p(x')}{p(x')} \quad \neg\mathcal{E}}{\perp} \quad \perp \\
 \frac{\perp}{\neg \forall x \neg p(x)} \quad \neg\mathcal{I}, -2 \\
 \frac{\neg \forall x \neg p(x)}{\exists x p(x) \supset \neg \forall x \neg p(x)} \quad \supset \mathcal{I}, -1
 \end{array}
 \end{array}$$

- Sekventsiesitus: lauseloogika loomuliku tuletuse sekventsiesituse reeglitele lisanduvad järgmised:

$$\frac{\Gamma \rightarrow A[y/x]}{\Gamma \rightarrow \forall x A} \forall\mathcal{I}^*$$

$$\frac{\Gamma \rightarrow \forall x A}{\Gamma \rightarrow A[t/x]} \forall\mathcal{E}$$

$$\frac{\Gamma \rightarrow A[t/x]}{\Gamma \rightarrow \exists x A} \exists\mathcal{I}$$

$$\frac{\Gamma \rightarrow \exists x A \quad \Gamma, A[y/x] \rightarrow C}{\Gamma \rightarrow C} \exists\mathcal{E}^\dagger$$

* y ei tohi vabalt esineda valemis $\forall x A$ ja valemihulgas Γ

† y ei tohi vabalt esineda valemites $\exists x A, C$ ja valemihulgas Γ

PREDIKAATLOOGIKA: SEKVENTSIARVUTUS

- Predikaatarvutuse sekventiarvutuses lisanduvad lauseloogika sekventsiarvutusele järgmised reeglid kvantorite kohta:

$$\begin{array}{c}
 \frac{\Gamma \rightarrow A[y/x], \Delta}{\Gamma \rightarrow \forall x A, \Delta} \forall \mathcal{R}^* \\
 \frac{\Gamma \rightarrow A[t/x], (\exists x A,) \Delta}{\Gamma \rightarrow \exists x A, \Delta} \exists \mathcal{R}
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\Gamma, (\forall x A,) A[t/x] \rightarrow \Delta}{\Gamma, \forall x A \rightarrow \Delta} \forall \mathcal{L} \\
 \frac{\Gamma, A[y/x] \rightarrow \Delta}{\Gamma, \exists x, A \rightarrow \Delta} \exists \mathcal{L}^\dagger
 \end{array}$$

* y ei tohi vabalt esineda valemis $\forall x A$ ja valemihulkades Γ, Δ

† y ei tohi vabalt esineda valemis $\exists x A$ ja valemihulkades Γ, Δ

- Näiteid tõestustest:

$$\begin{array}{c}
 \frac{}{p(x', y') \rightarrow p(x', y')} \text{ hyp.} \\
 \frac{}{\forall y p(x', y) \rightarrow p(x', y')} \forall \mathcal{L} \\
 \frac{}{\forall y p(x', y) \rightarrow \exists x p(x, y')} \exists \mathcal{R} \\
 \frac{}{\exists x \forall y p(x, y) \rightarrow \exists x p(x, y')} \exists \mathcal{L}, x' \text{ värske} \\
 \frac{}{\exists x \forall y p(x, y) \rightarrow \forall y \exists x p(x, y)} \forall \mathcal{R}, y' \text{ värske} \\
 \hline
 \rightarrow \exists x \forall y p(x, y) \supset \forall y \exists x p(x, y) \supset \mathcal{R}
 \end{array}$$

$$\begin{array}{c}
 \frac{}{p(x') \rightarrow p(x')} \text{ hyp.} \\
 \frac{}{p(x'), \neg p(x') \rightarrow} \neg \mathcal{L} \\
 \frac{}{p(x'), \forall x \neg p(x) \rightarrow} \forall \mathcal{L} \\
 \frac{}{\exists x p(x), \forall x \neg p(x) \rightarrow} \exists \mathcal{L}, x' \text{ värske} \\
 \frac{}{\exists x p(x) \rightarrow \neg \forall x \neg p(x)} \neg \mathcal{R} \\
 \hline
 \rightarrow \exists x p(x) \supset \neg \forall x \neg p(x) \supset \mathcal{R}
 \end{array}$$

$$\begin{array}{c}
\frac{}{p(f(x'), y') \rightarrow p(f(x'), y')} \text{ hyp.} \\
\frac{}{p(y', g(x')) \rightarrow p(y', g(x'))} \text{ hyp.} \\
\frac{}{p(f(x'), y') \rightarrow \exists y p(f(x'), y)} \exists \mathcal{R} \\
\frac{}{p(y', g(x')) \rightarrow \exists y p(y', y)} \exists \mathcal{R} \\
\frac{}{p(f(x'), y') \rightarrow \exists x \exists y p(x, y)} \exists \mathcal{R} \\
\frac{}{p(y', g(x')) \rightarrow \exists x \exists y p(x, y)} \exists \mathcal{R} \\
\frac{}{p(f(x'), y') \vee p(y', g(x')) \rightarrow \exists x \exists y p(x, y)} \vee \mathcal{L} \\
\frac{}{\exists y (p(f(x'), y) \vee p(y, g(x')))} \exists \mathcal{L}, y' \text{ v\u00e4rske} \\
\frac{}{\exists x \exists y (p(f(x), y) \vee p(y, g(x)))} \exists \mathcal{L}, x' \text{ v\u00e4rske} \\
\frac{}{\rightarrow \exists x \exists y (p(f(x), y) \vee p(y, g(x)))} \supset \mathcal{R}
\end{array}$$