Multi-Party Computation in Presence of Corrupted Majorities

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on joint work with
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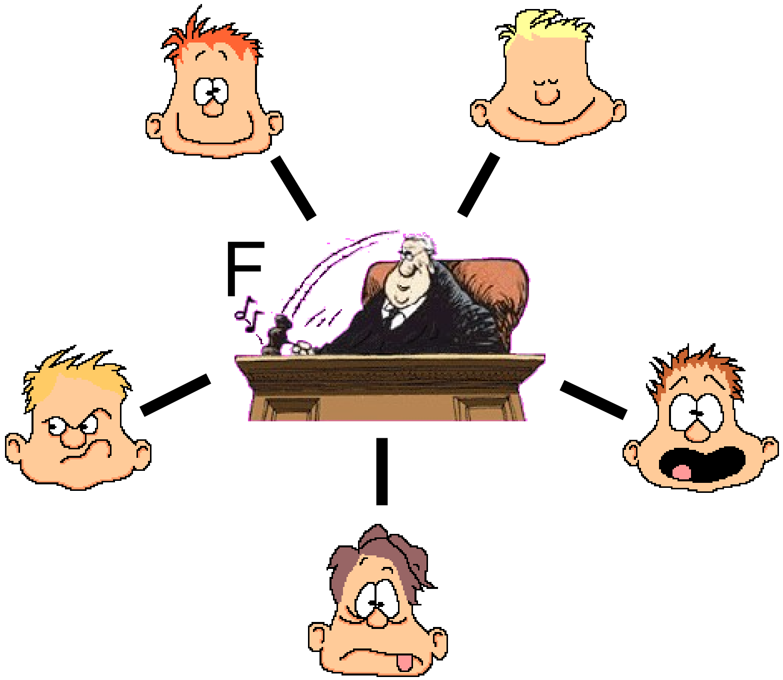
Mäetaguse, 2009/10/04

Multi-Party Computation (MPC)



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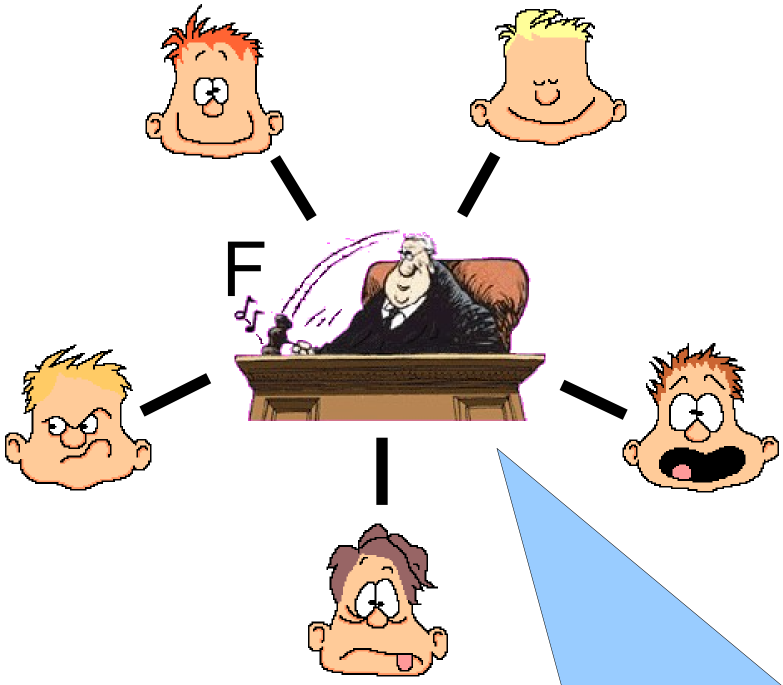

- Voting
- Auctions
- Who is richest?

⇒ privacy, correctness required

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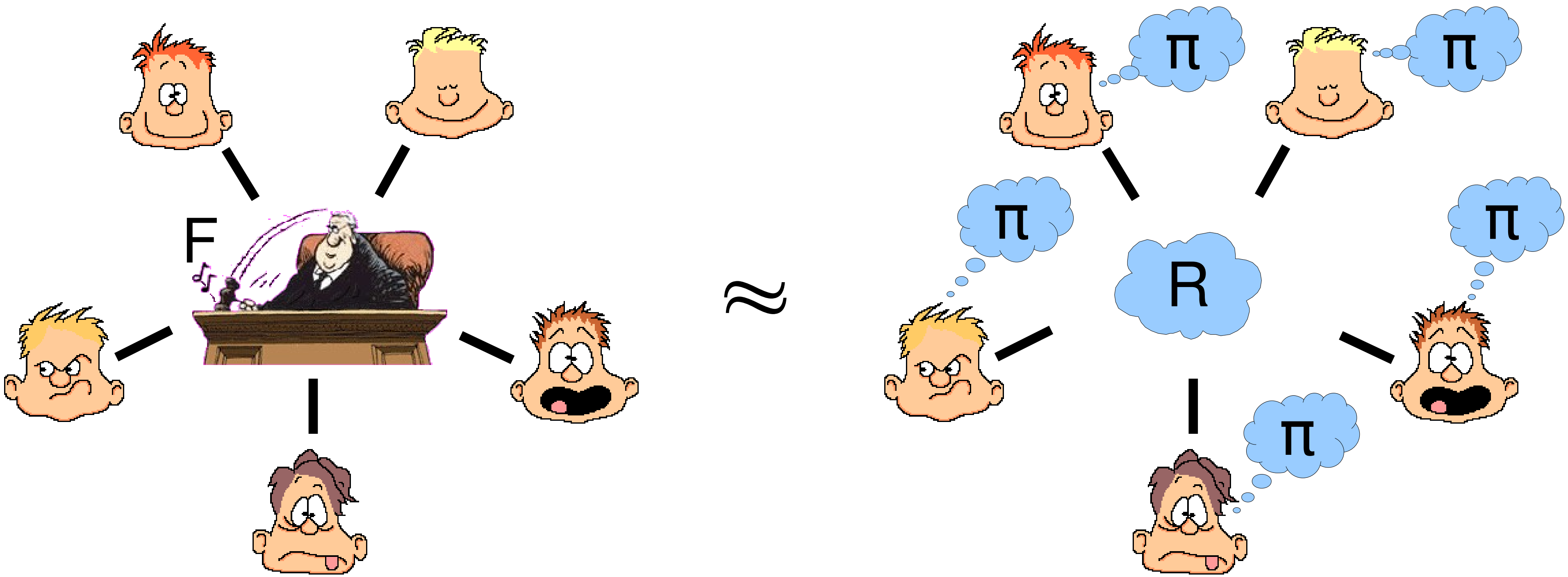

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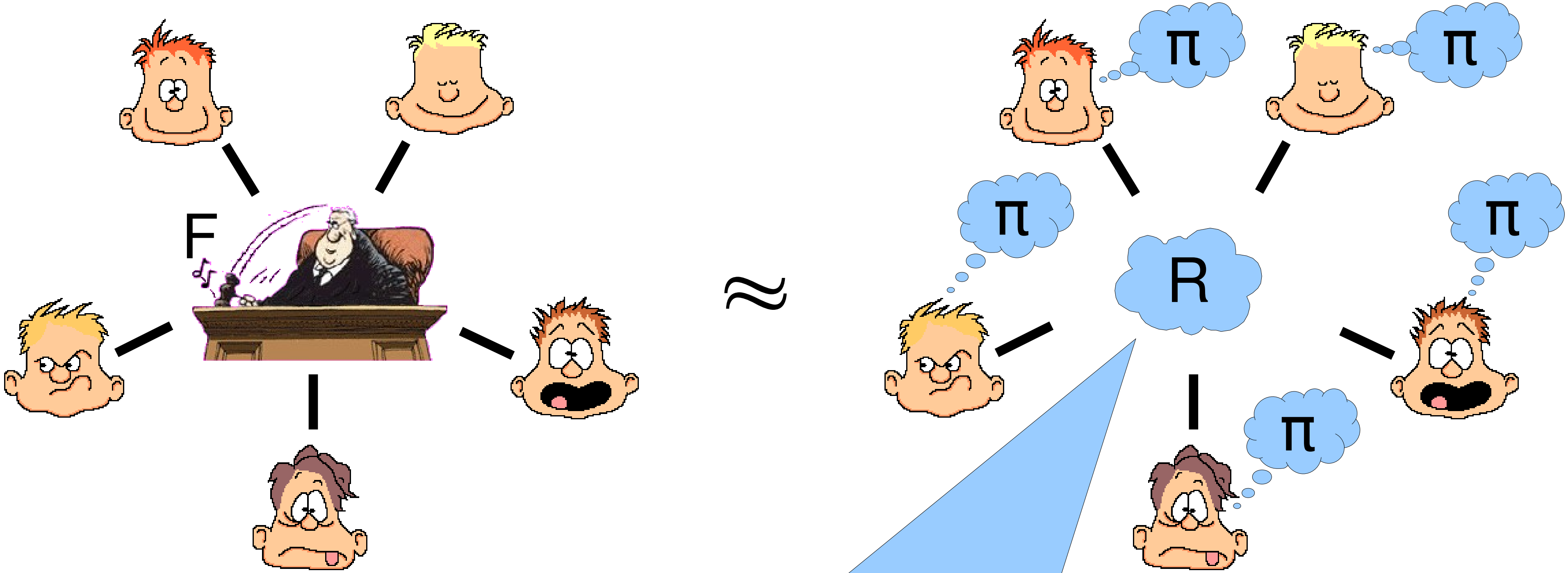

Generally encompasses:

- Secure or authenticated channels
- Optionally BC or PKI
- CRS for UC setting

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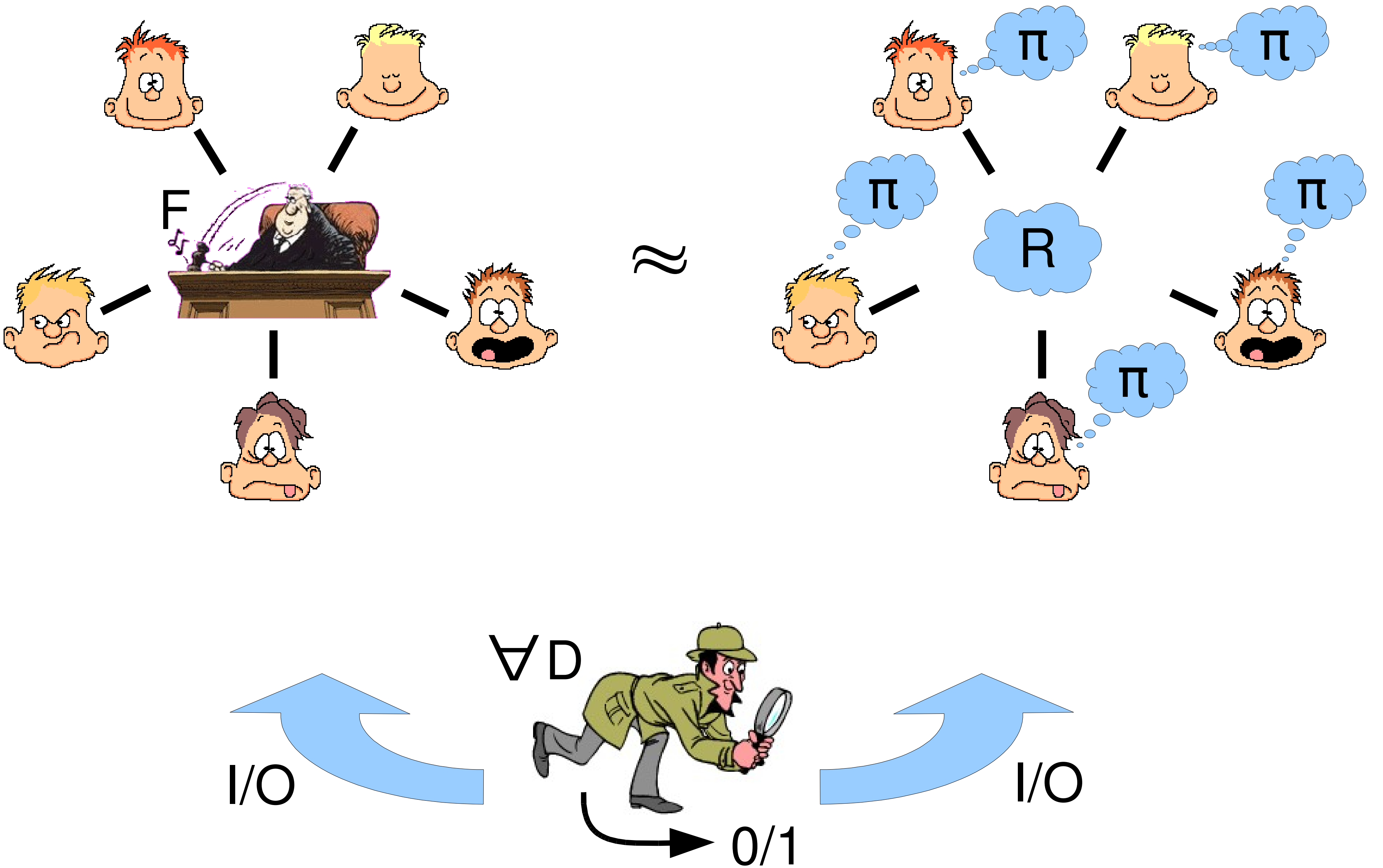
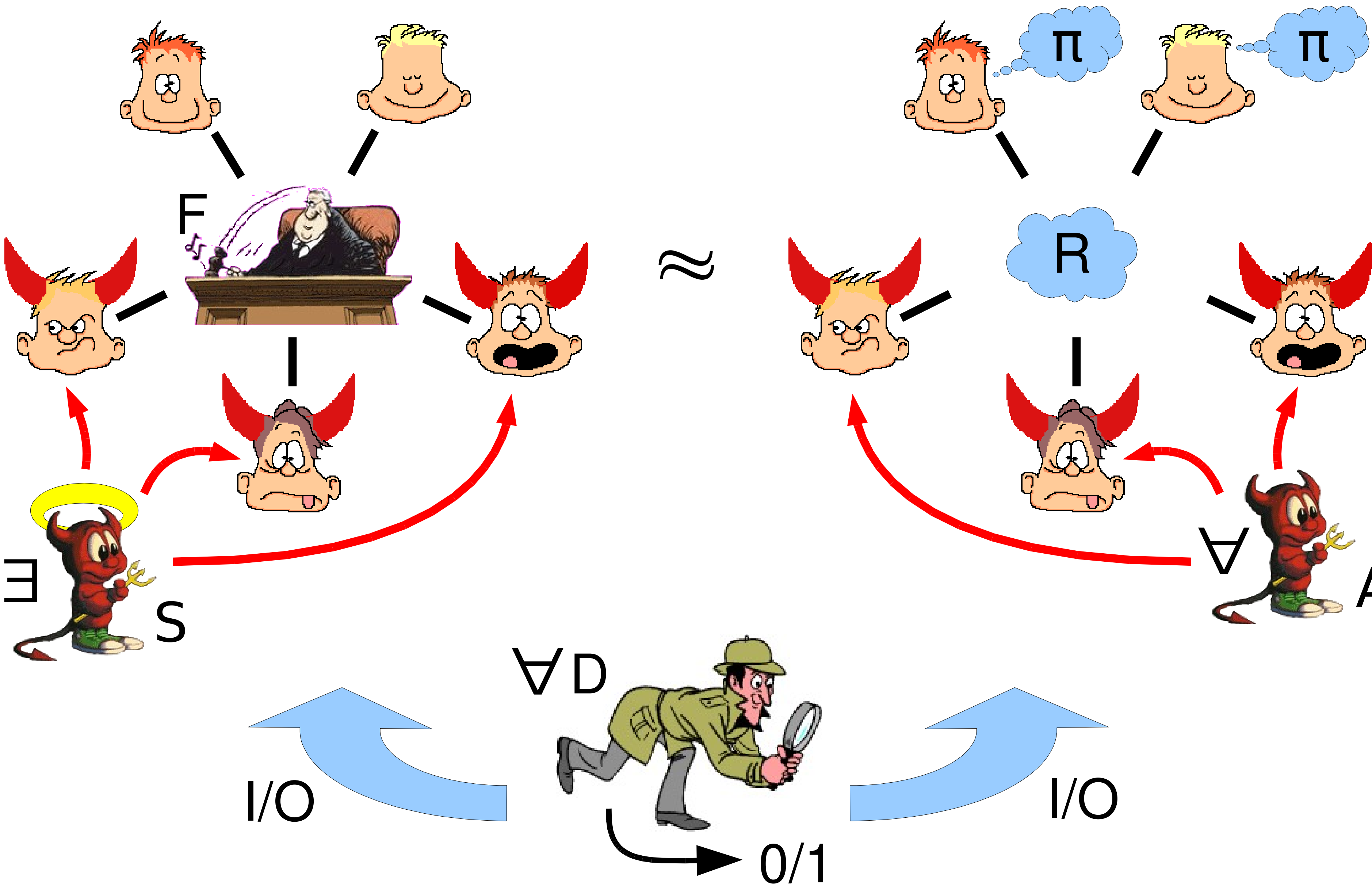
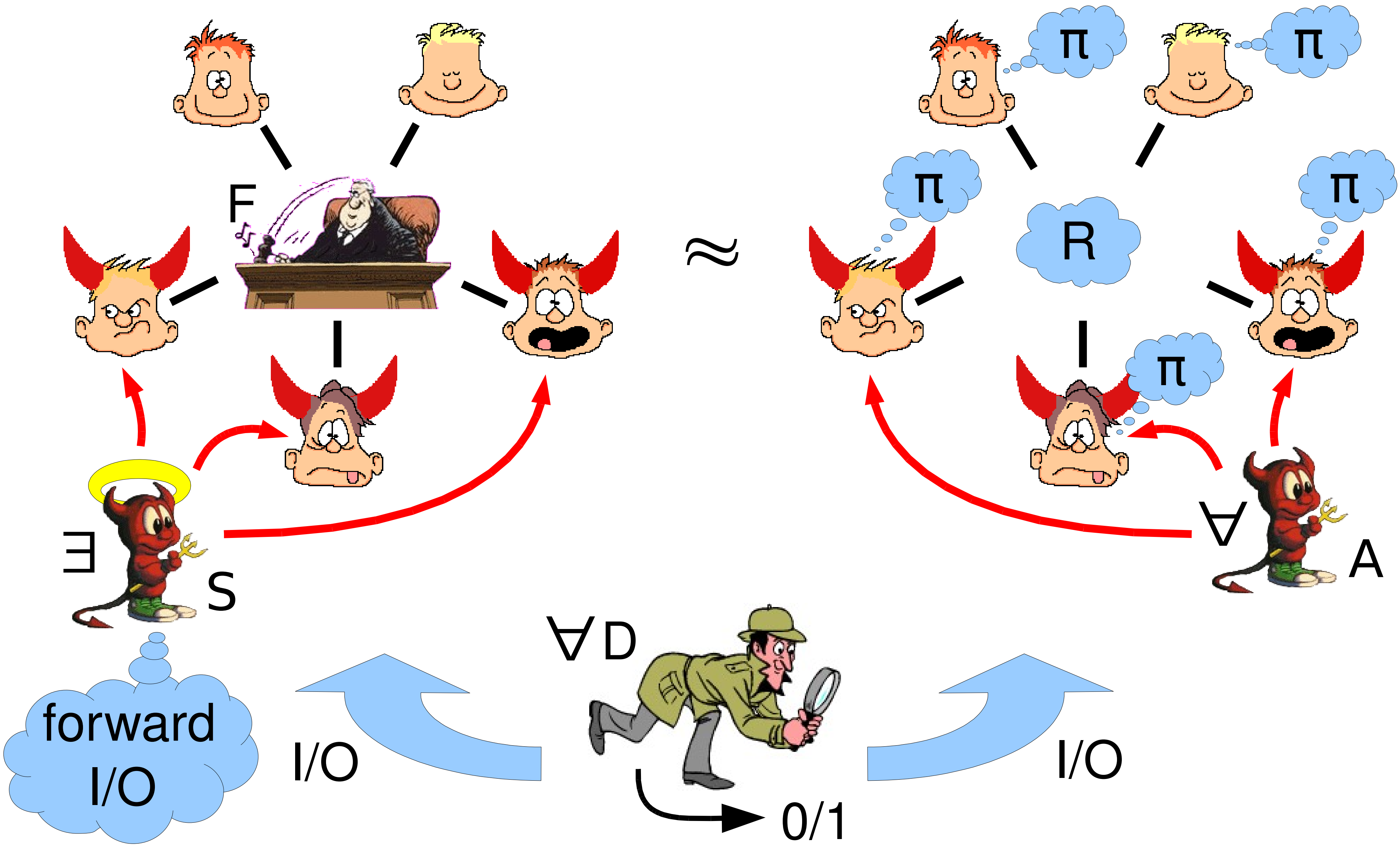


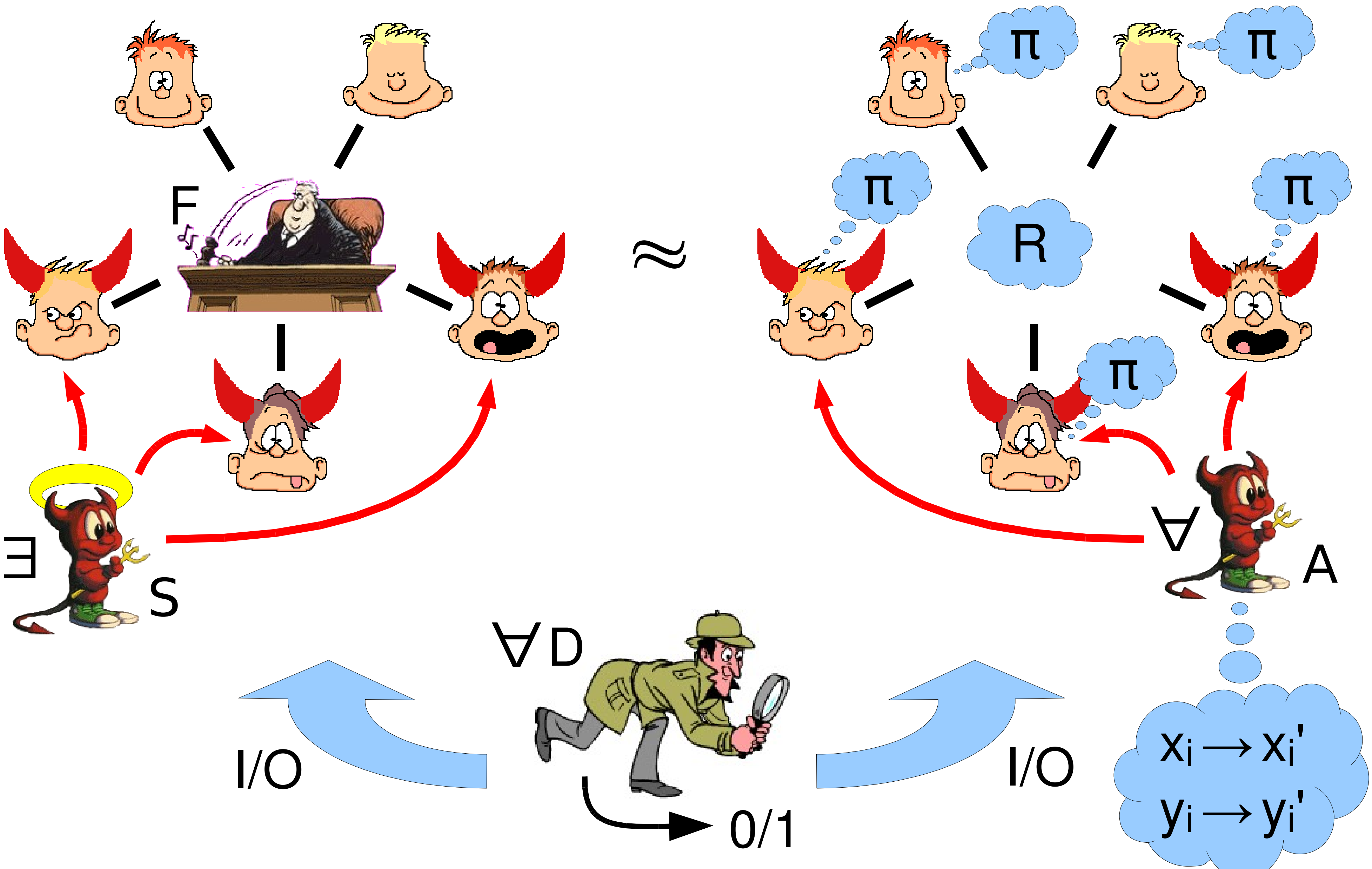
MPC: Active Adversary



MPC: Passive Adversary



MPC: Semi-Honest Adversary



Security Properties for MPC

- Correctness: protocol computes intended result
- Privacy: nobody learns more than intended
- Robustness: everybody receives intended result
- Fairness: everybody receives result, or nobody
- Agreement (on abort): all honest parties receive their result or notification of failure

Security Paradigms for MPC

- Abort Security: agreement, privacy, correctness
- Fair Security: fairness, privacy, correctness
- Full Security: robustness, privacy, correctness

- IT Security: tolerates unbounded adversaries
- CO Security: tolerates computationally bounded adversaries

Limitations for MPC with BC

- Fair security only for $t < n/2$ corrupted [Cle86]
- IT security only for $t < n/2$ [Kil00]
- Full security for t_1 and abort security for t_2 only if $t_1 + t_2 < n$ [IKLP06], [Kat07]

- No IT full security for general MPC for $t \geq n/2$
  - ⇒ Which functions can be computed with IT full security for $t \geq n/2$?
  - ⇒ Weaker assumptions, graceful degradation?

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  $\Rightarrow$ Which functions can be computed with IT full security for $t \geq n/2$?

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Computability of Functions

Security	Adversary	Resources	Fair?	Computable f
IT	passive	auth. BC	yes	F^{bc}_{pas}
	semi-honest	auth. BC	yes	F^{bc}_{sh}
	active	auth. BC	yes	F^{bc}_{act}

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- Today: only symmetric functions
- Then: $F^{bc}_{sh} = F^{bc}_{pas}$

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LT	active	auth. BC	no	F^{bc}_{lts}
		auth. chan.	no	F^{aut}_{lts}
		PKI	no	$F^{ins,pki}_{lts}$

- Long-term (LT) security
  - Computational assumptions only during protocol run

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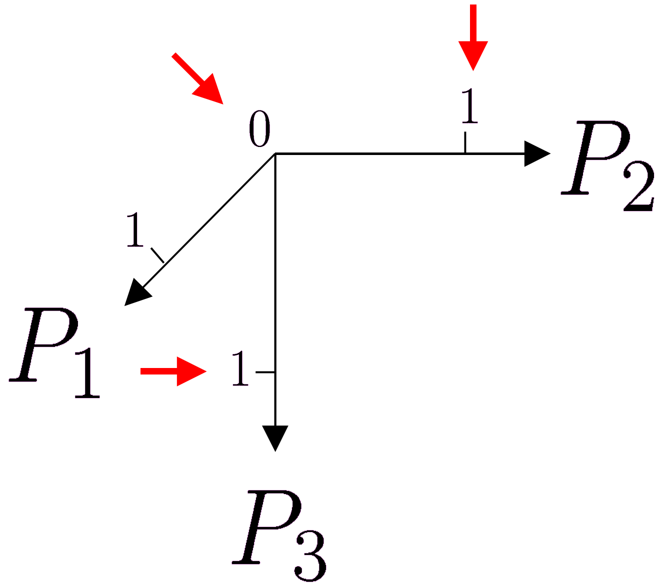
- “=”: modified [GMW87]-Compiler
  - computationally forces semi-honest behavior
  - maintains IT security against semi-honest adversary

Passively Computable Functions F^{bc}_{pas}



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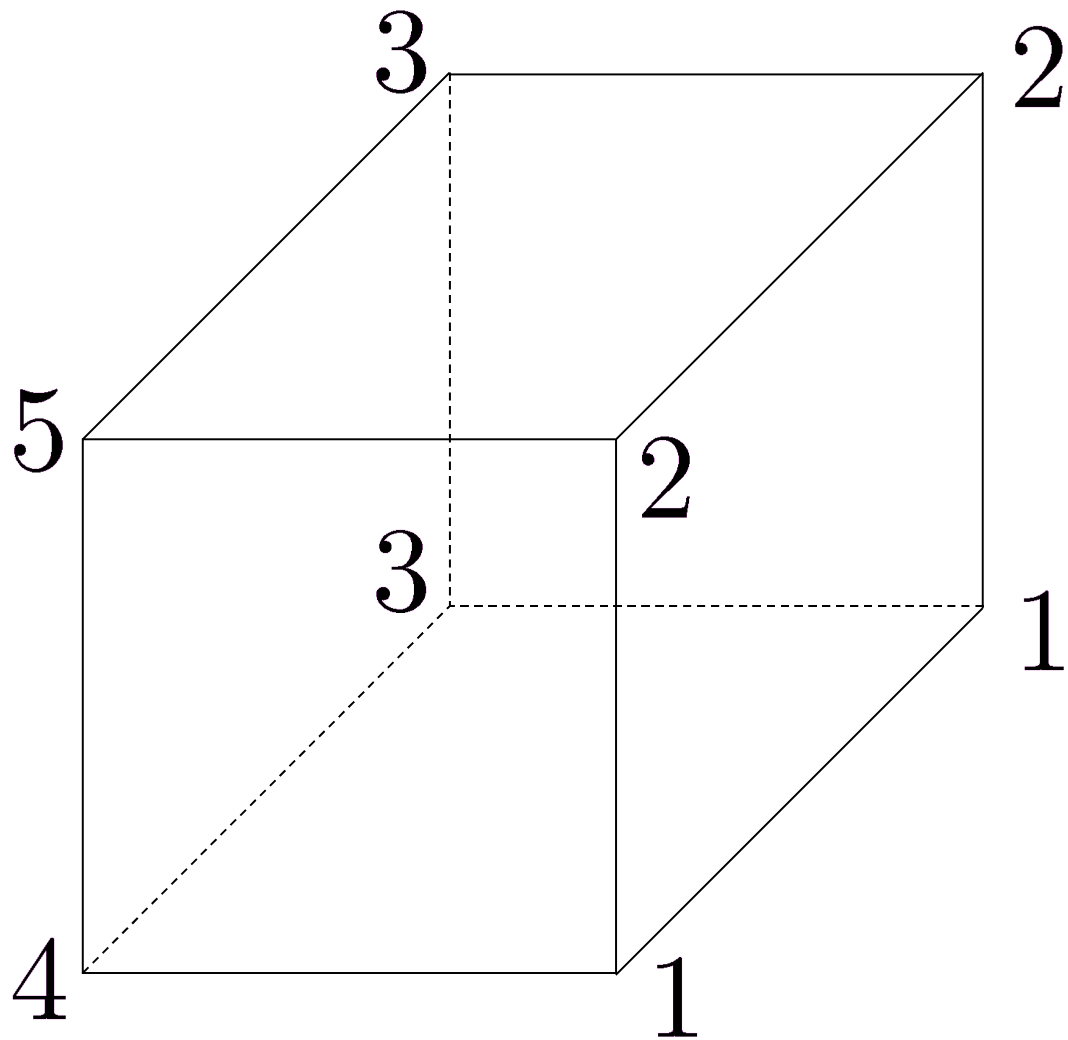
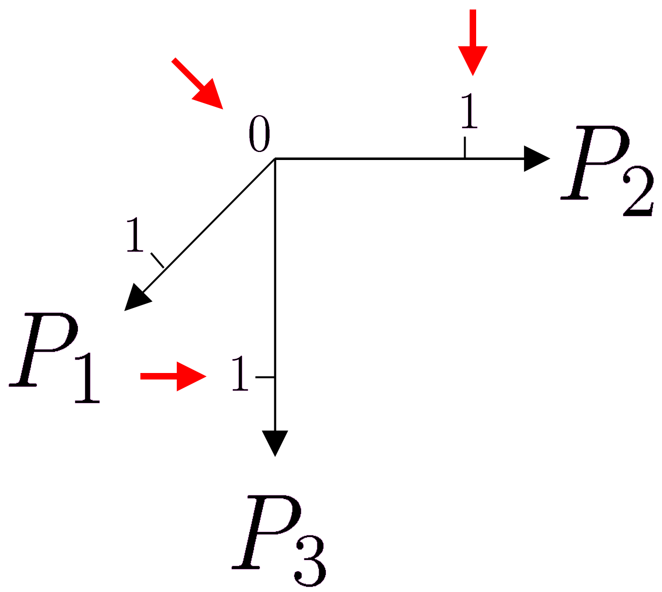


Passively Computable Functions F_{pas}^{bc}



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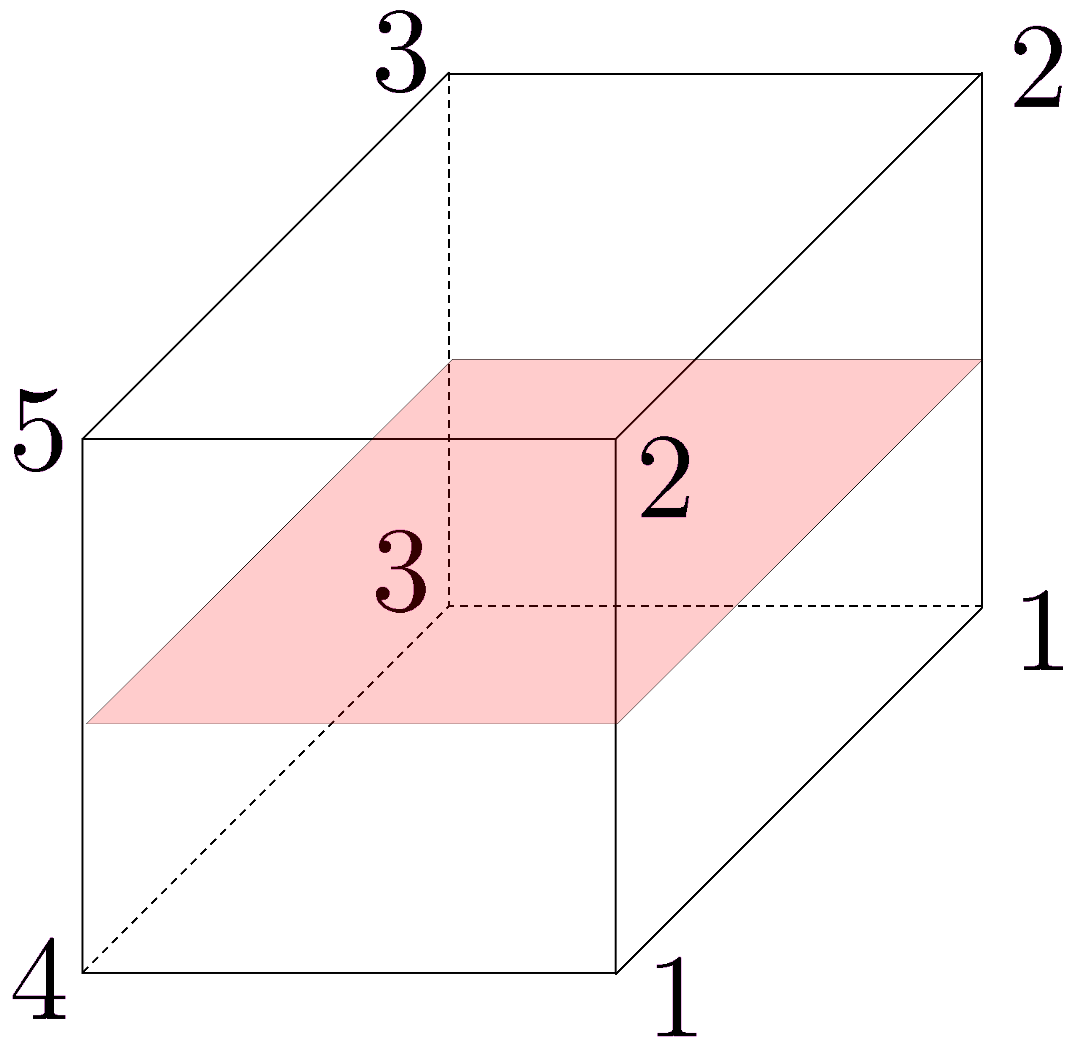

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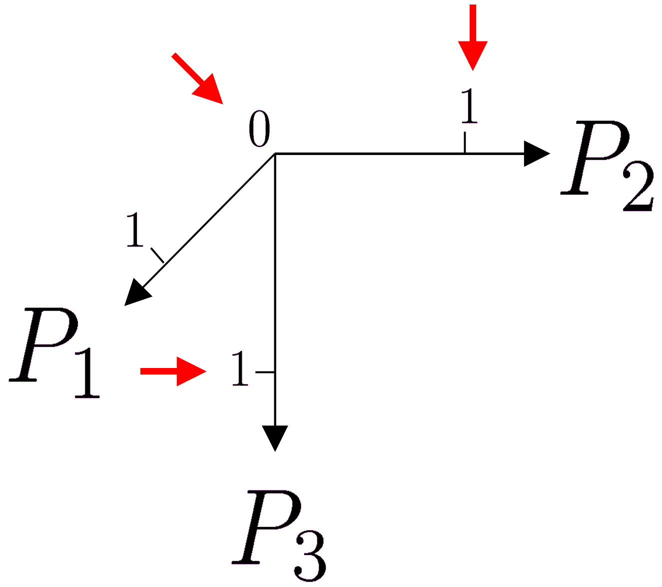

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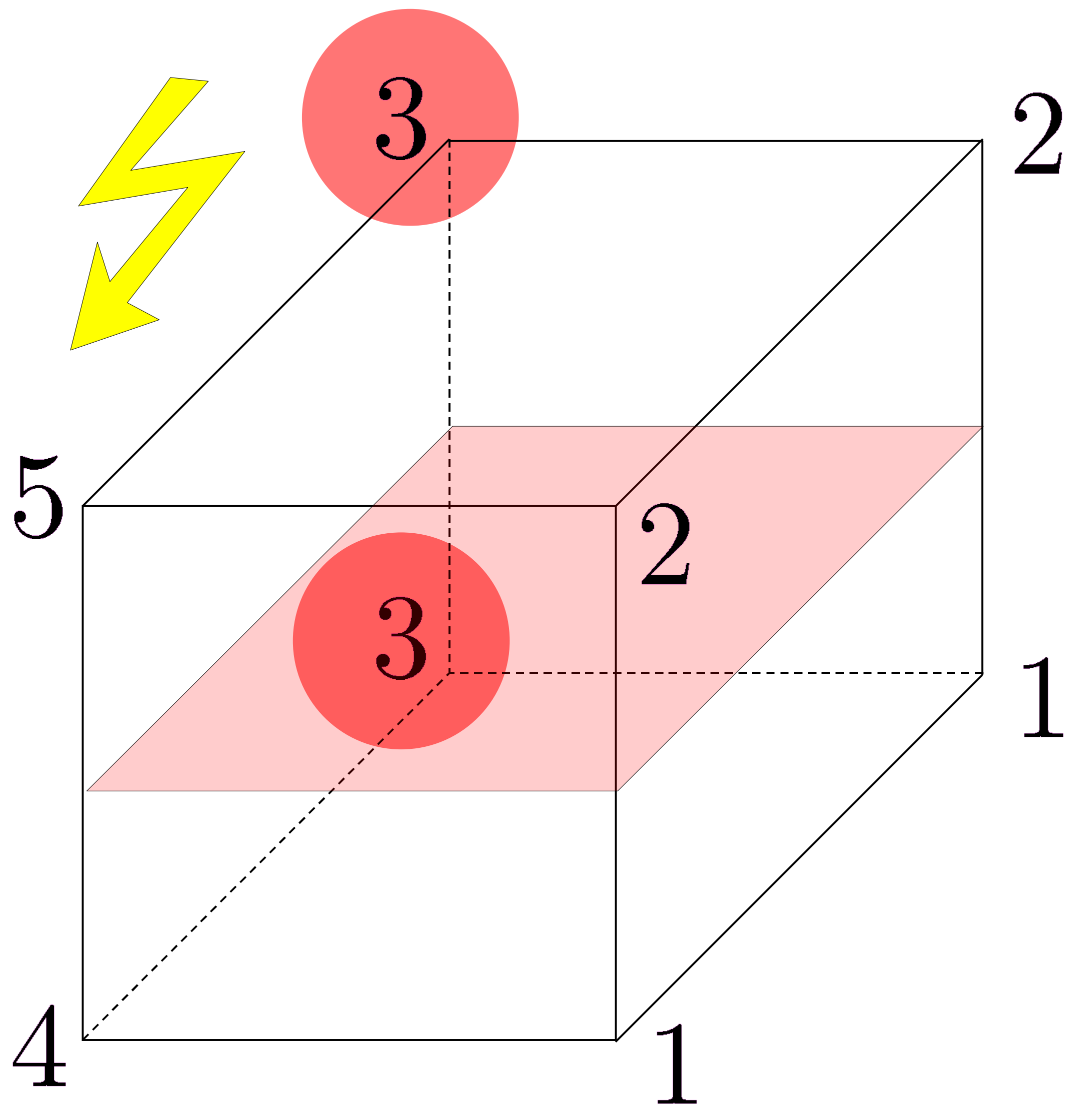

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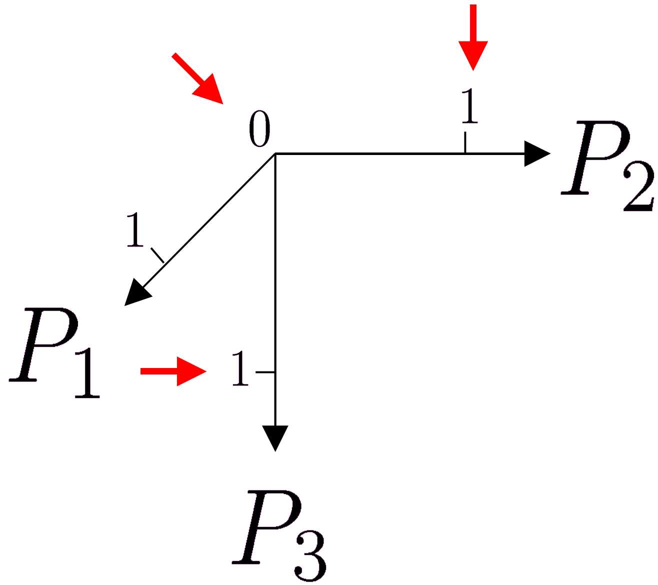

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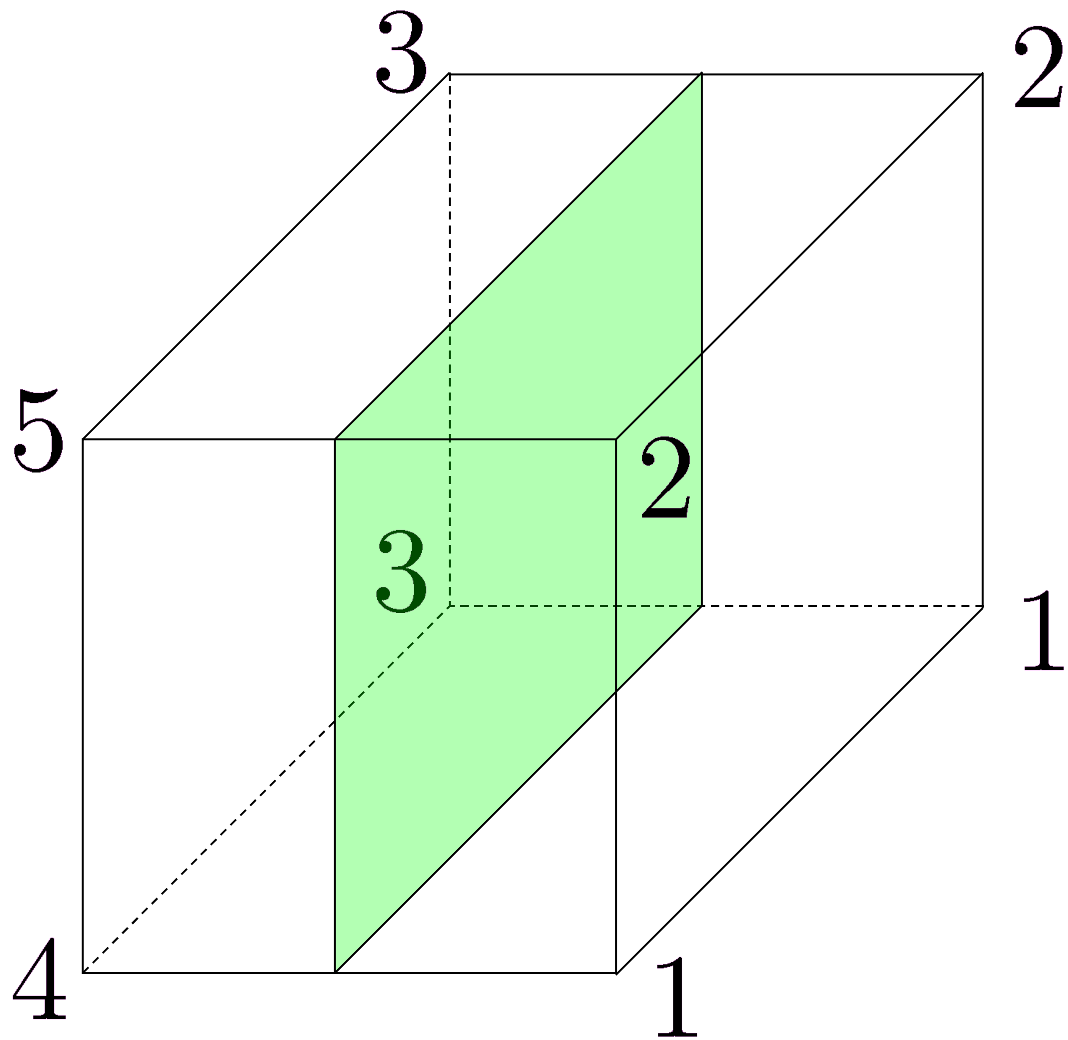

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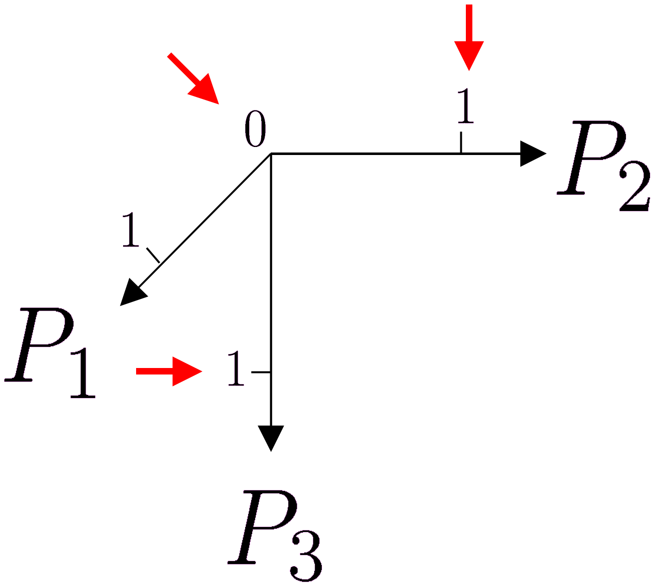

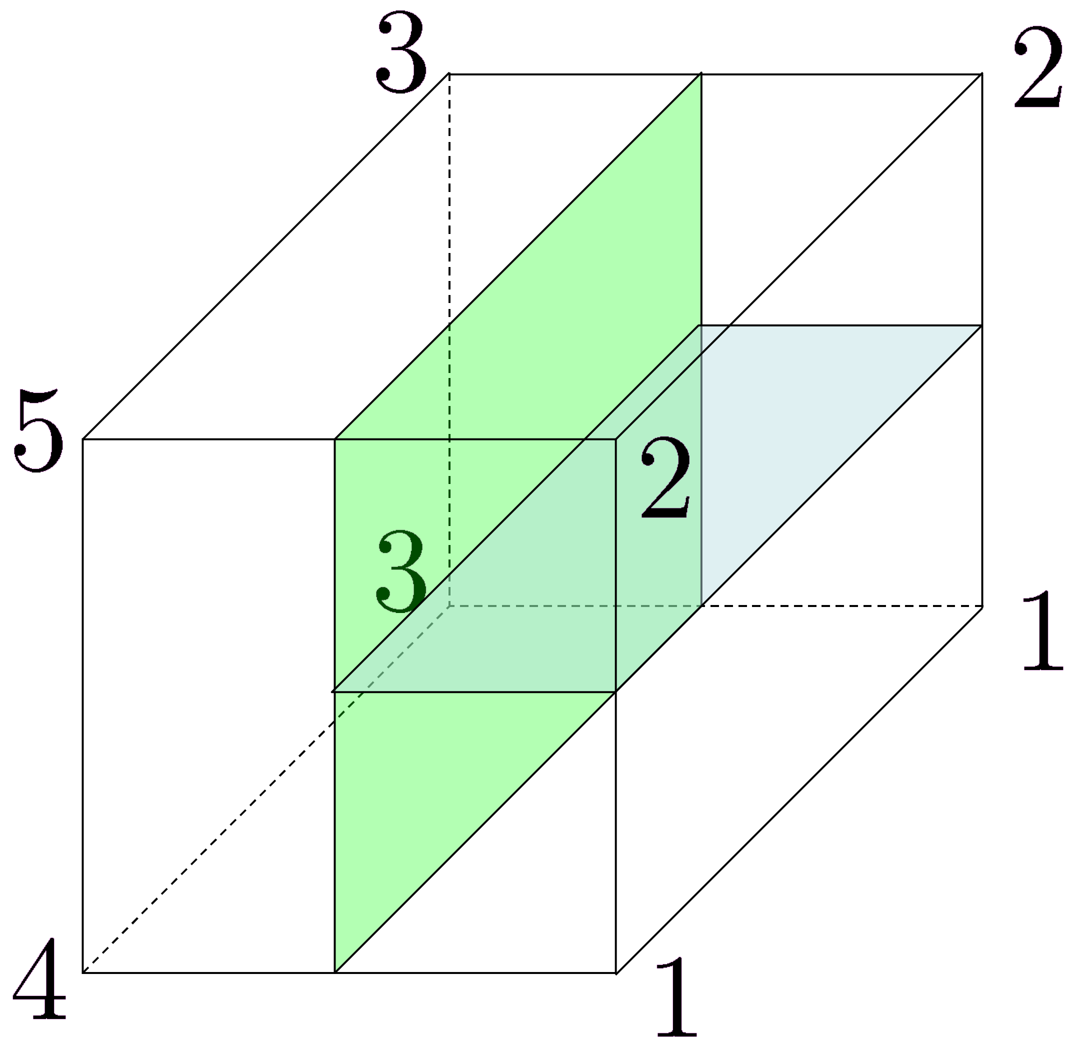

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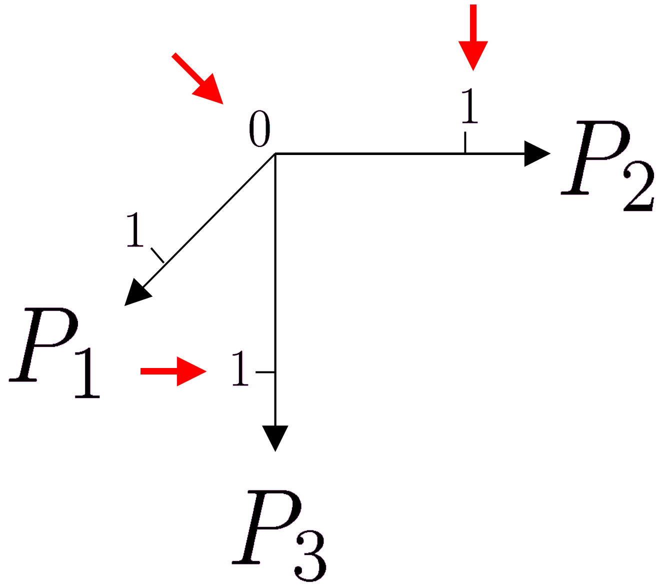

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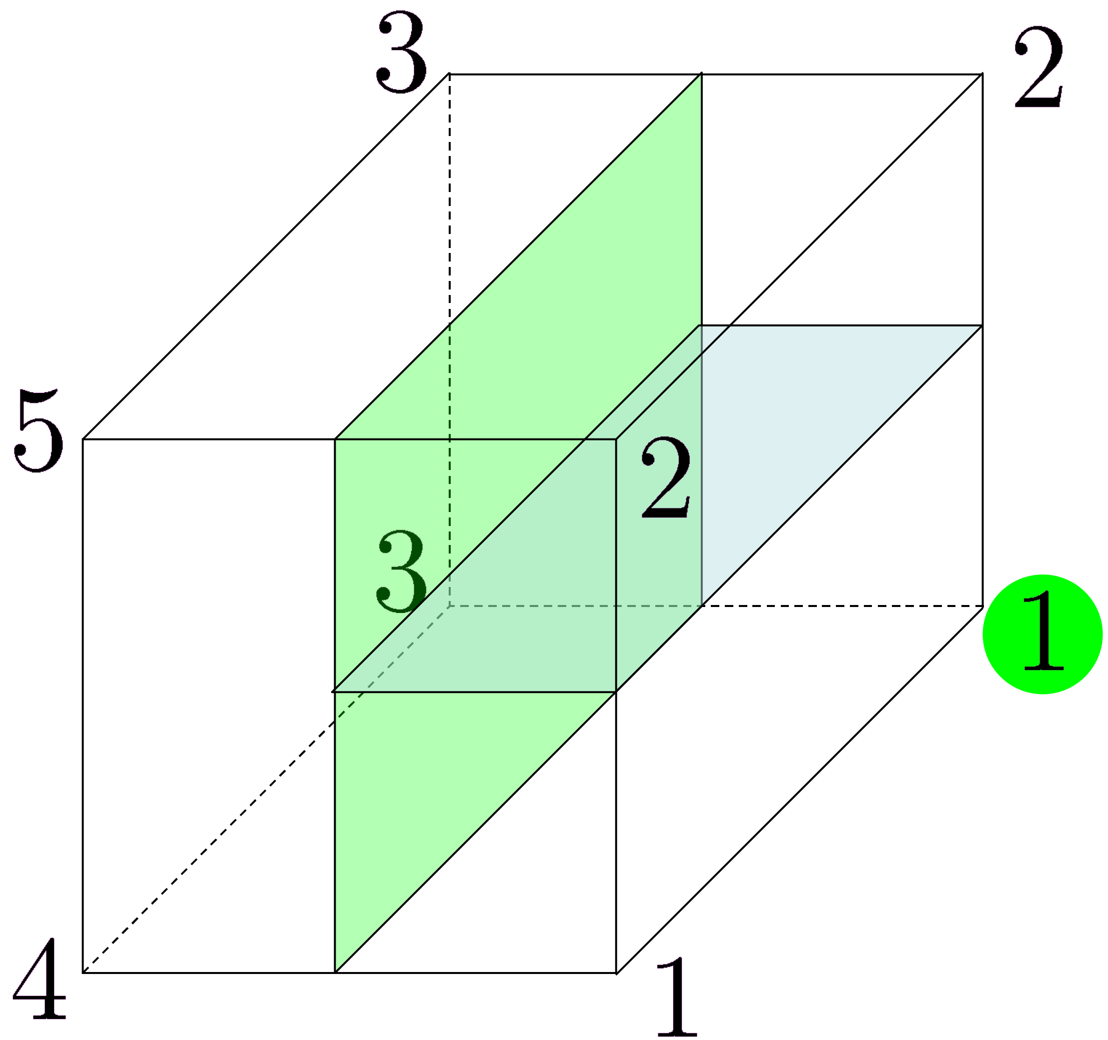

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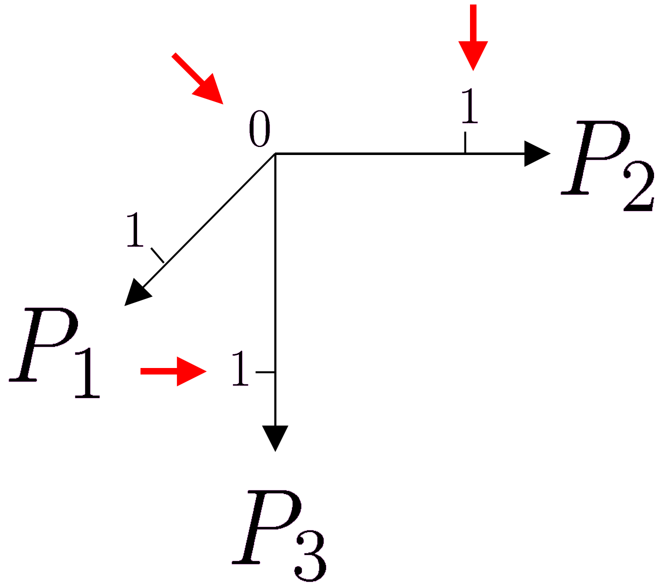

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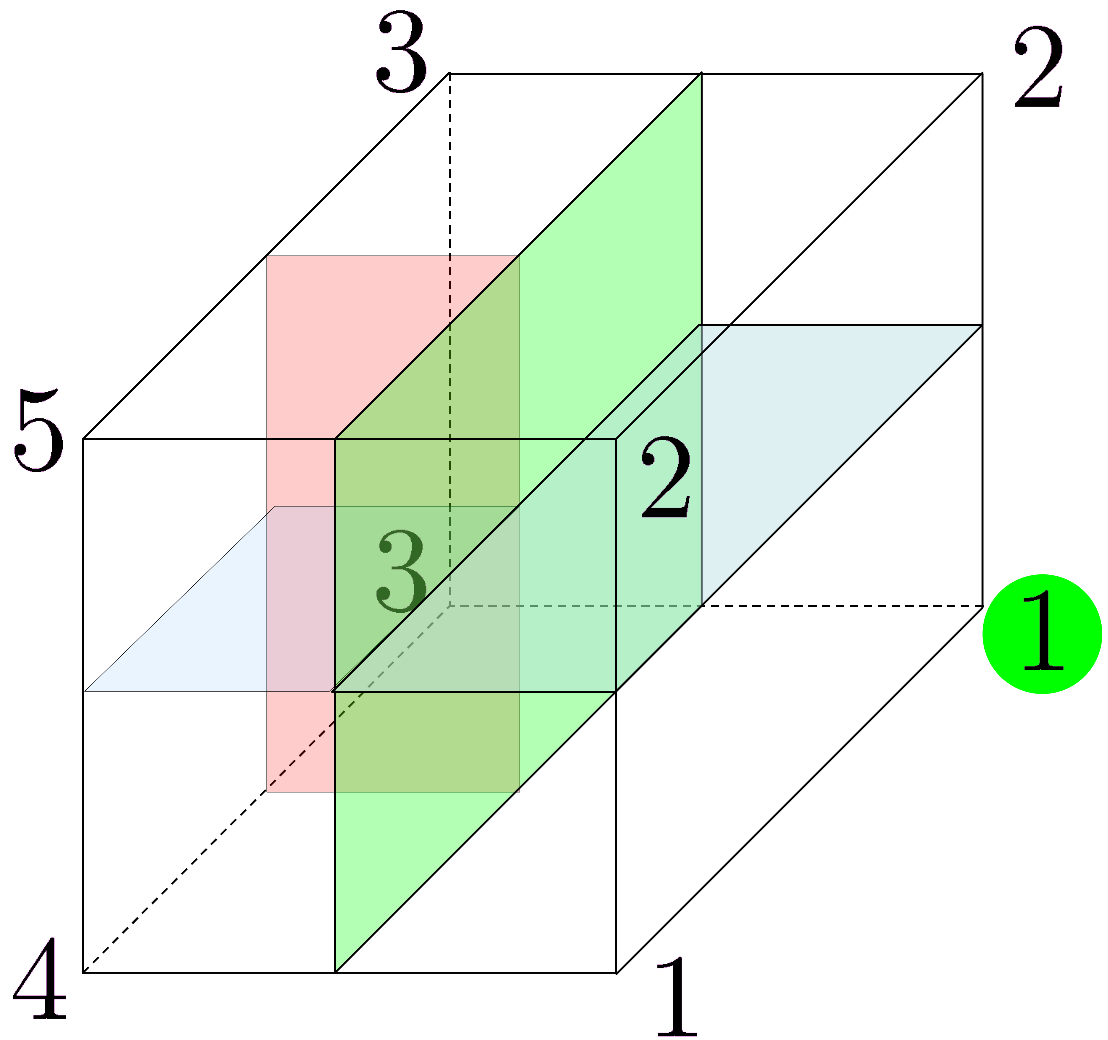
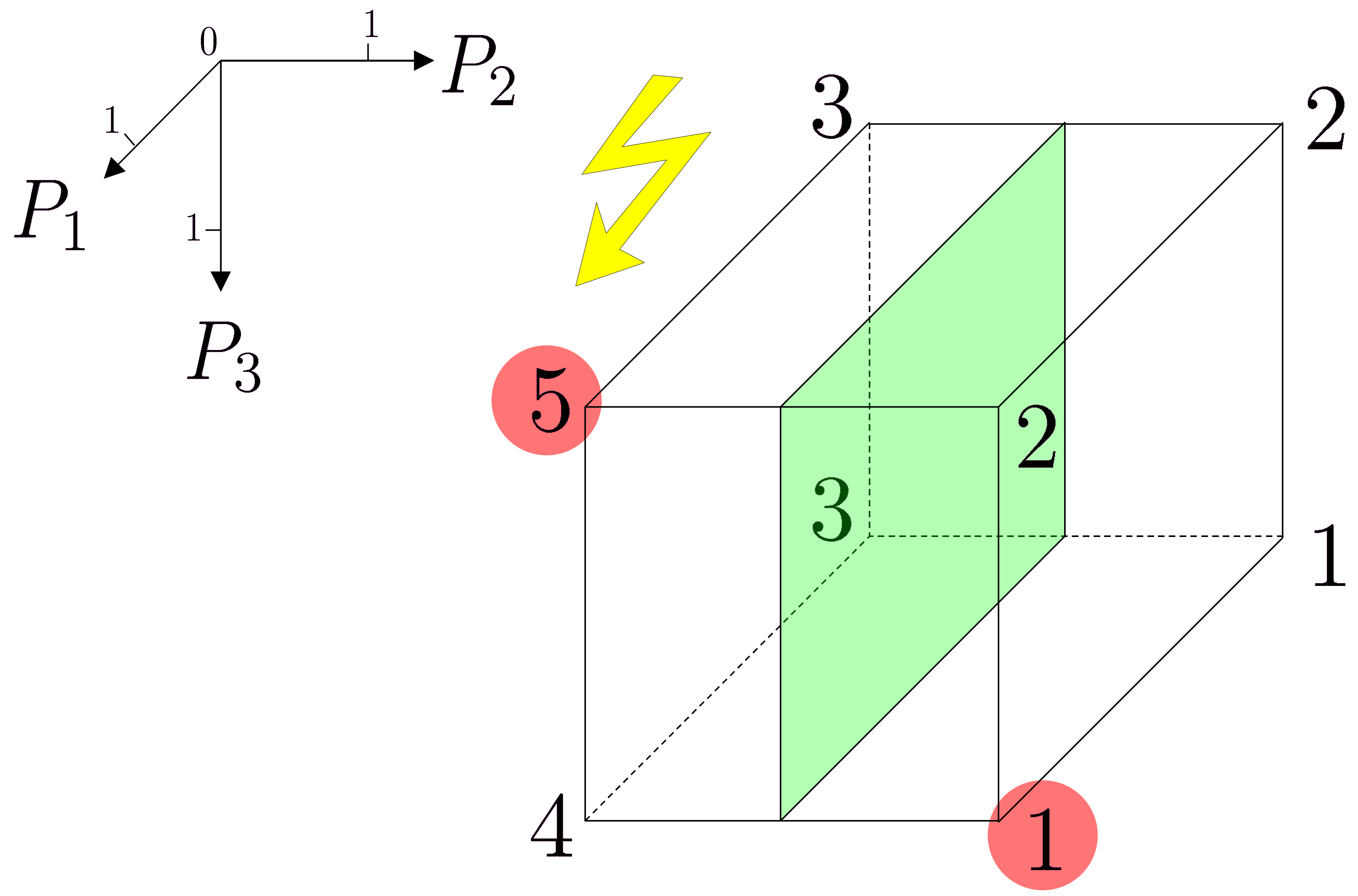


Actively Computable Functions F^{bc}_{act}



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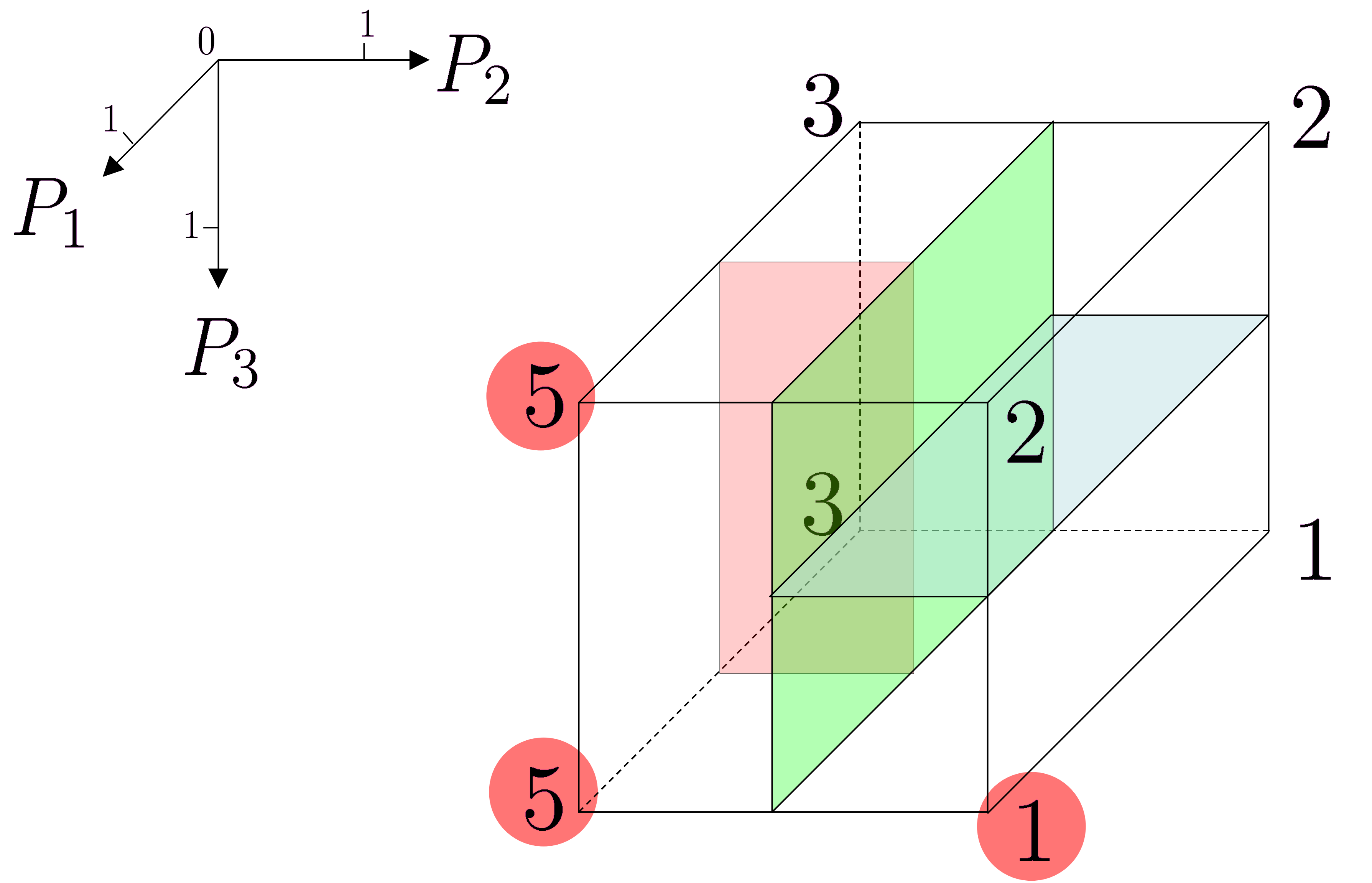
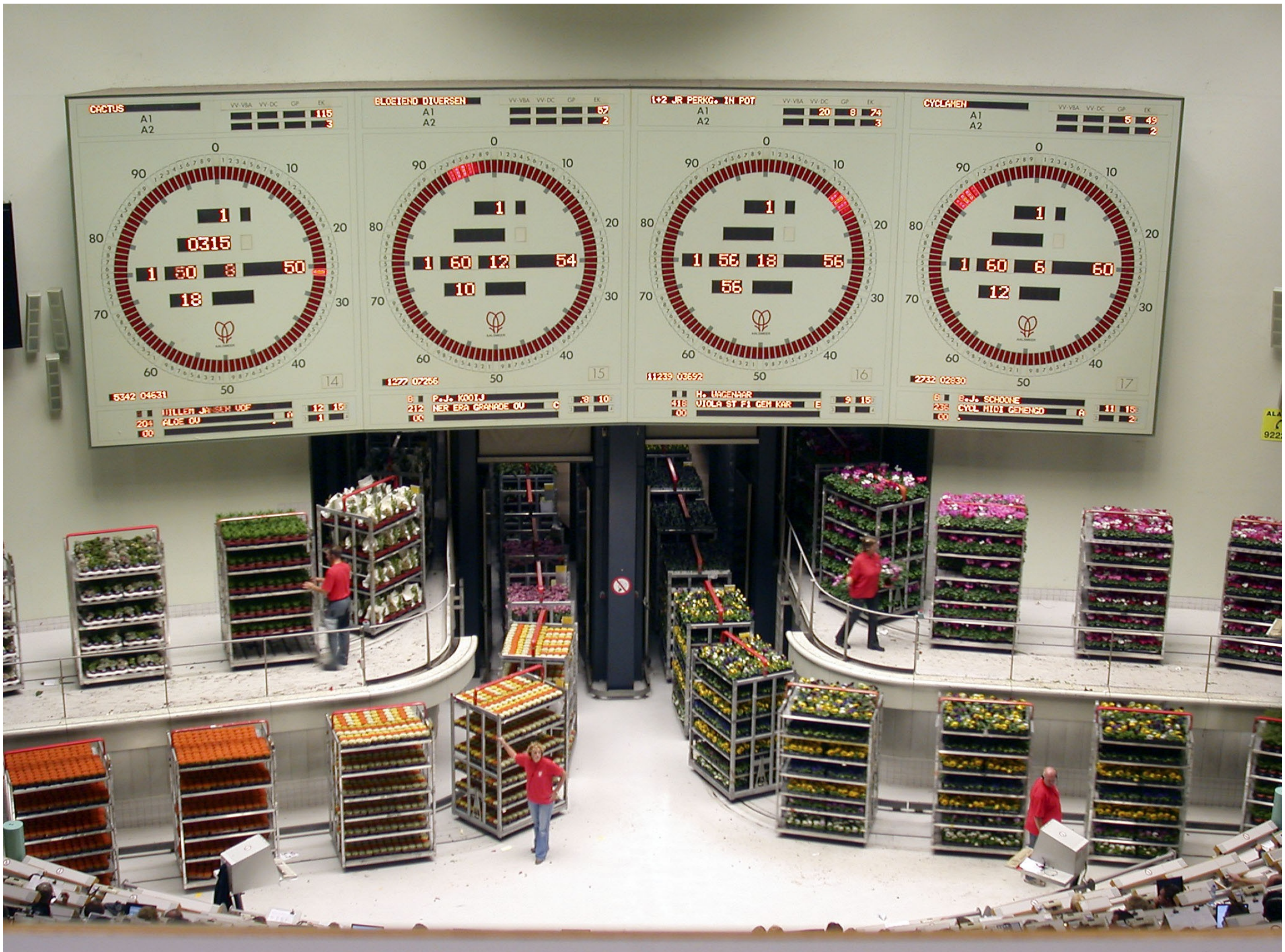


Actively Computable: Example



Summary: Computability

- Characterization of computable function classes
  - F^{bc}_{pas} : decomposability
  - F^{bc}_{sh} : decomposability after removing redundancy
  - F^{bc}_{act} : decomposability after removing redundancy, exchange property (input for every strategy)
- Characterization of long-term security:

$$F^{ins,pki}_{lts} = F^{aut}_{lts} = F^{bc}_{lts} = F^{bc}_{sh}$$

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  ⇒ Hybrid-secure MPC (HMPC)

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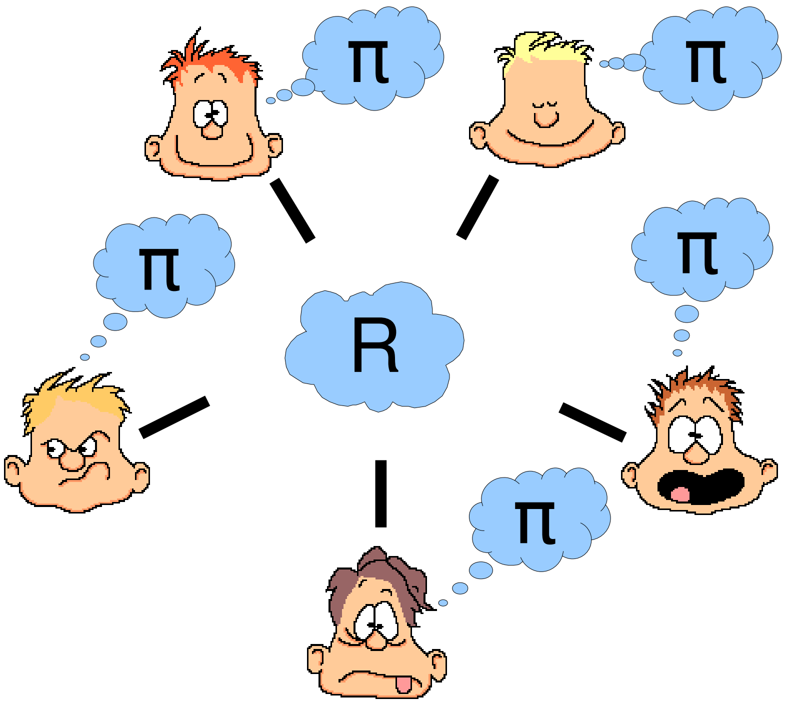
Optimal Hybrid MPC (with BC)



Goal: For any ρ < n/2

- IT full security for t ≤ ρ
- IT fair security for t < n/2
- CO abort security for t < n-ρ

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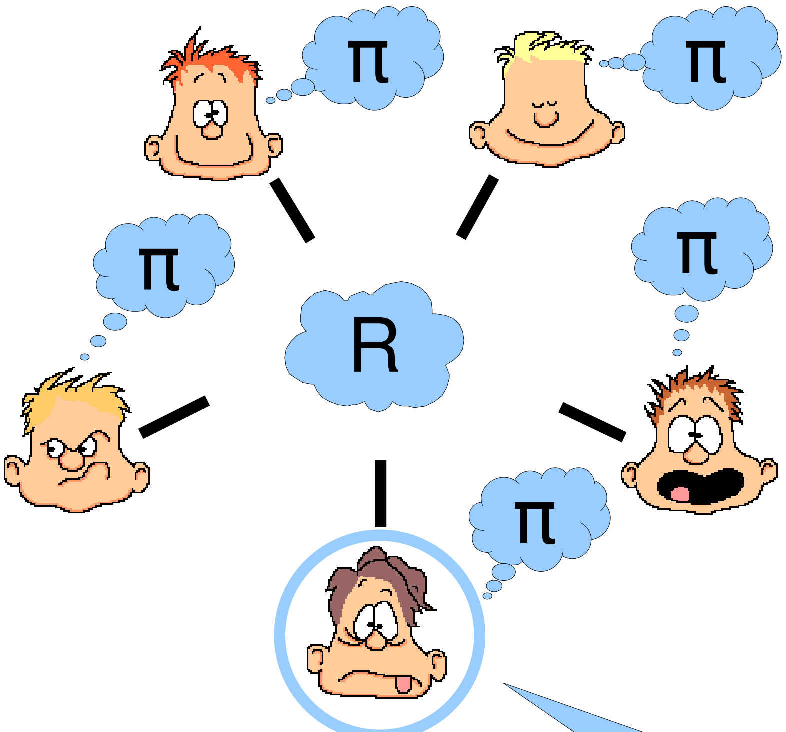

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[GMW87], [CLOS01]: can be IT protected

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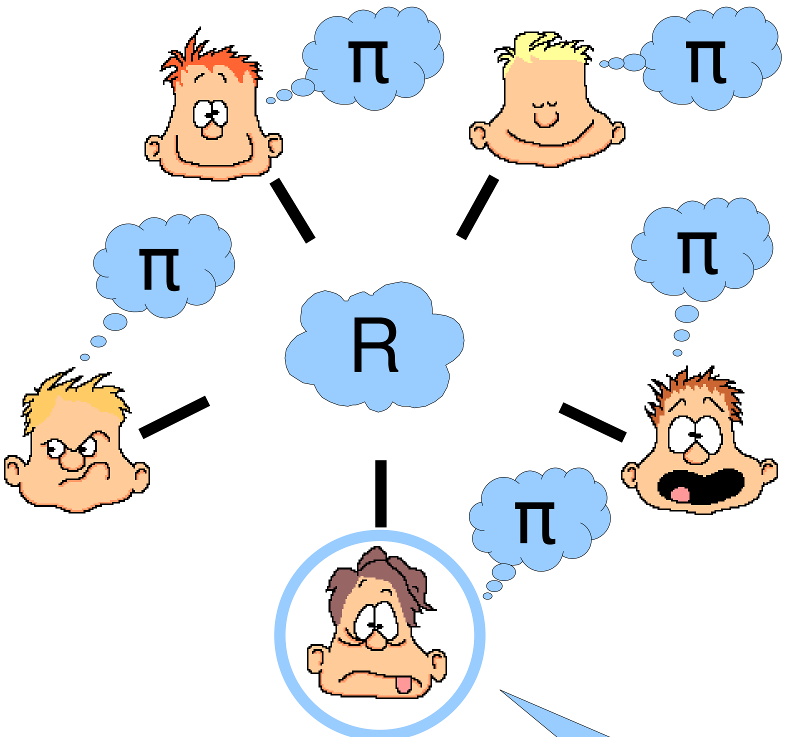

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Trusted ⇒
IT fairness, correctness

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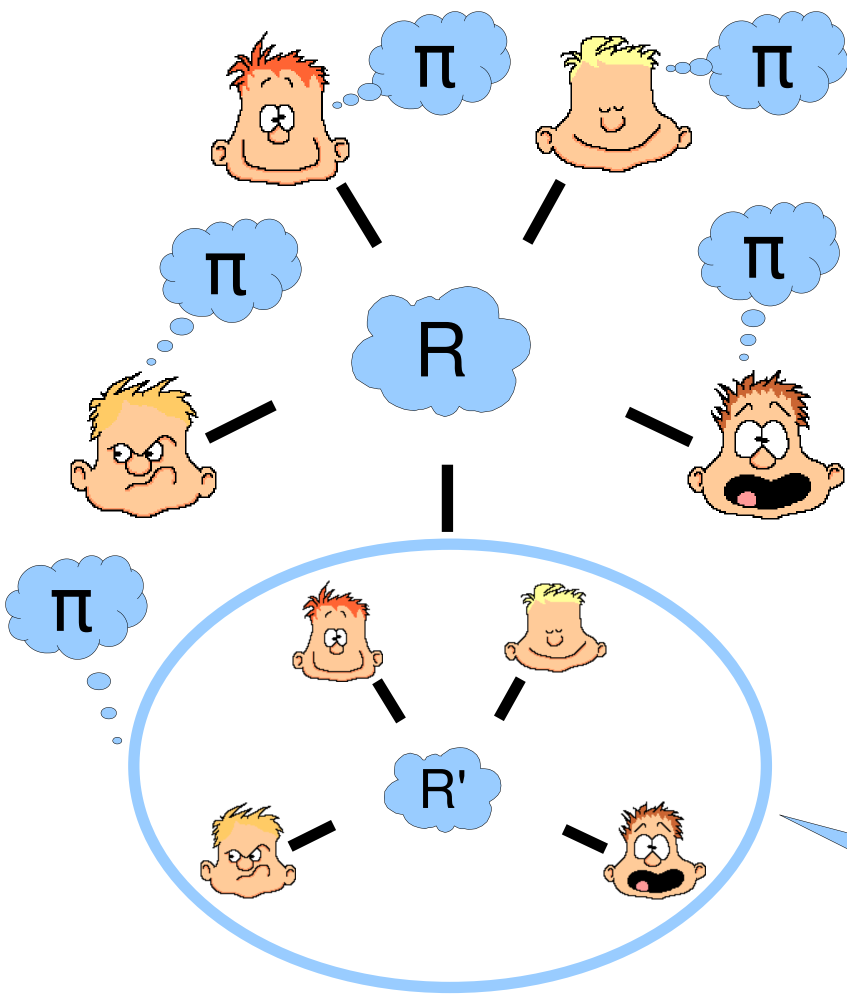

[Cha89]: emulate!

⇒ honest for t < n/2 [RB89]

⇒ t < n/2: IT fair, correct

⇒ t ≥ n/2: CO private, correct

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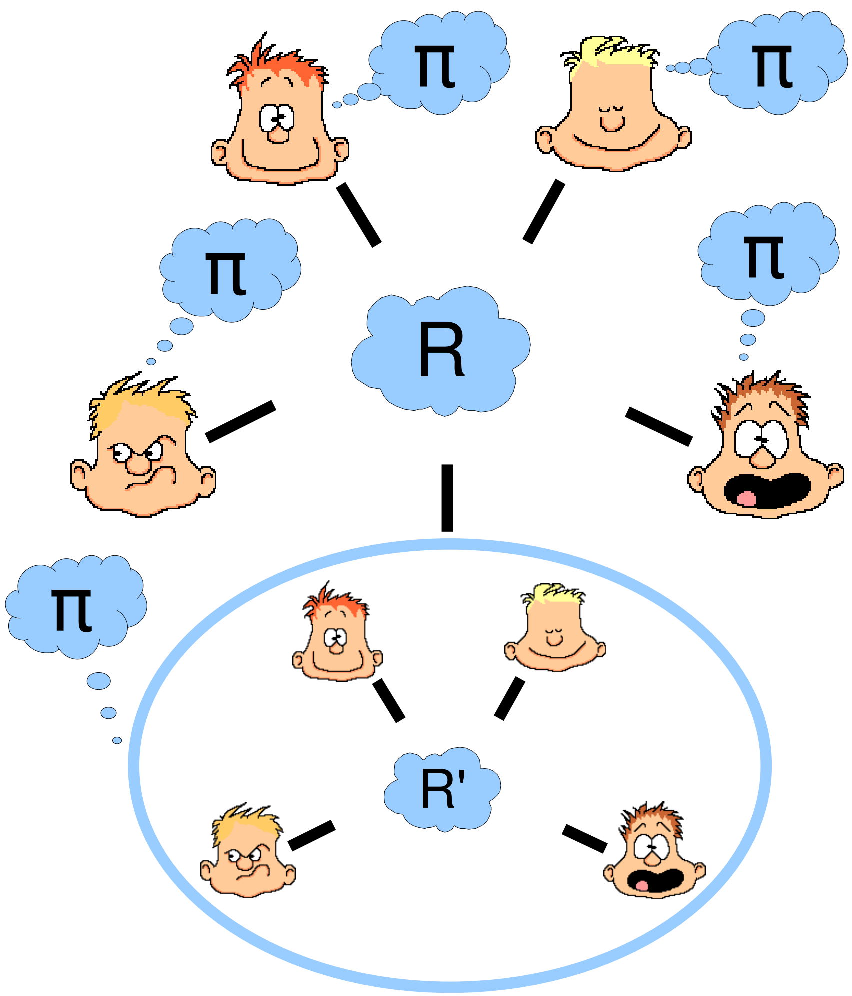


[Cha89]: emulate!

⇒ honest for $t < n/2$ [RB89]

⇒ $t < n/2$: IT fair, correct

⇒ $t \geq n/2$: CO private, correct

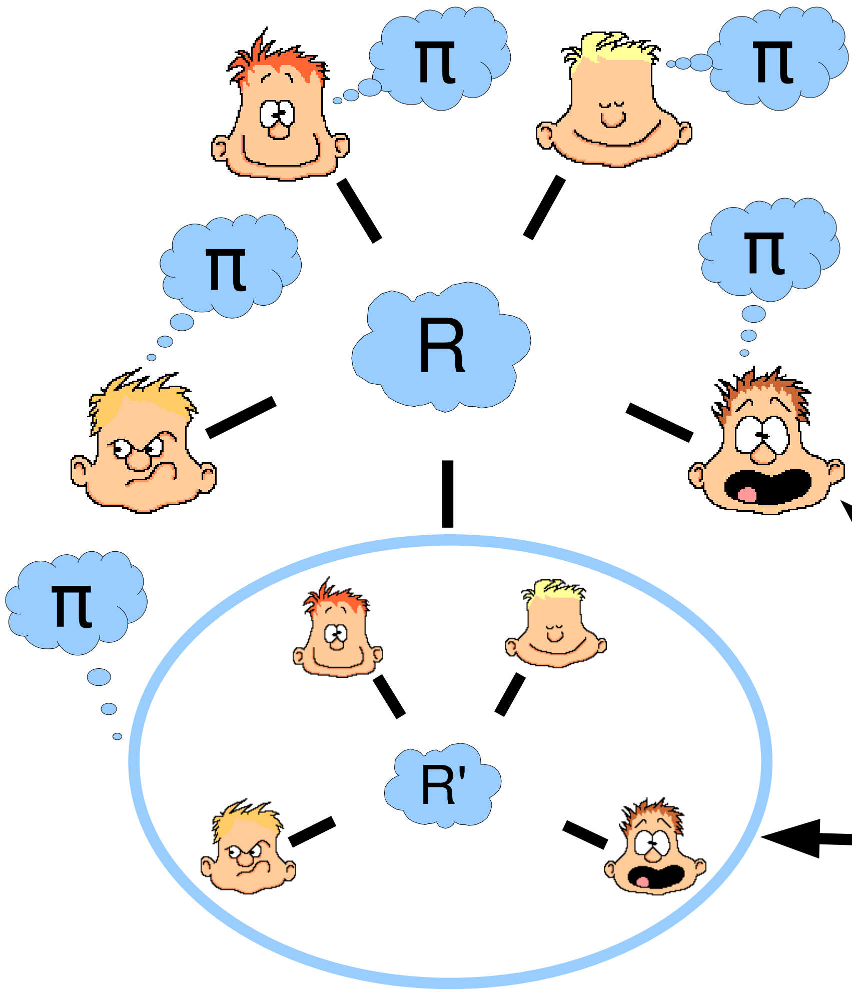
Use sharing qualifying all sets of emulated and n-ρ actual parties

⇒ $t \leq \rho$: IT robust, correct

⇒ $t < n/2$: IT fair, correct

⇒ $t < n-\rho$: CO private, correct

Optimal Hybrid MPC (with BC)



Share inputs

⇒ $t < n/2$: IT privacy

⇒ $t \geq n/2$: no correctness

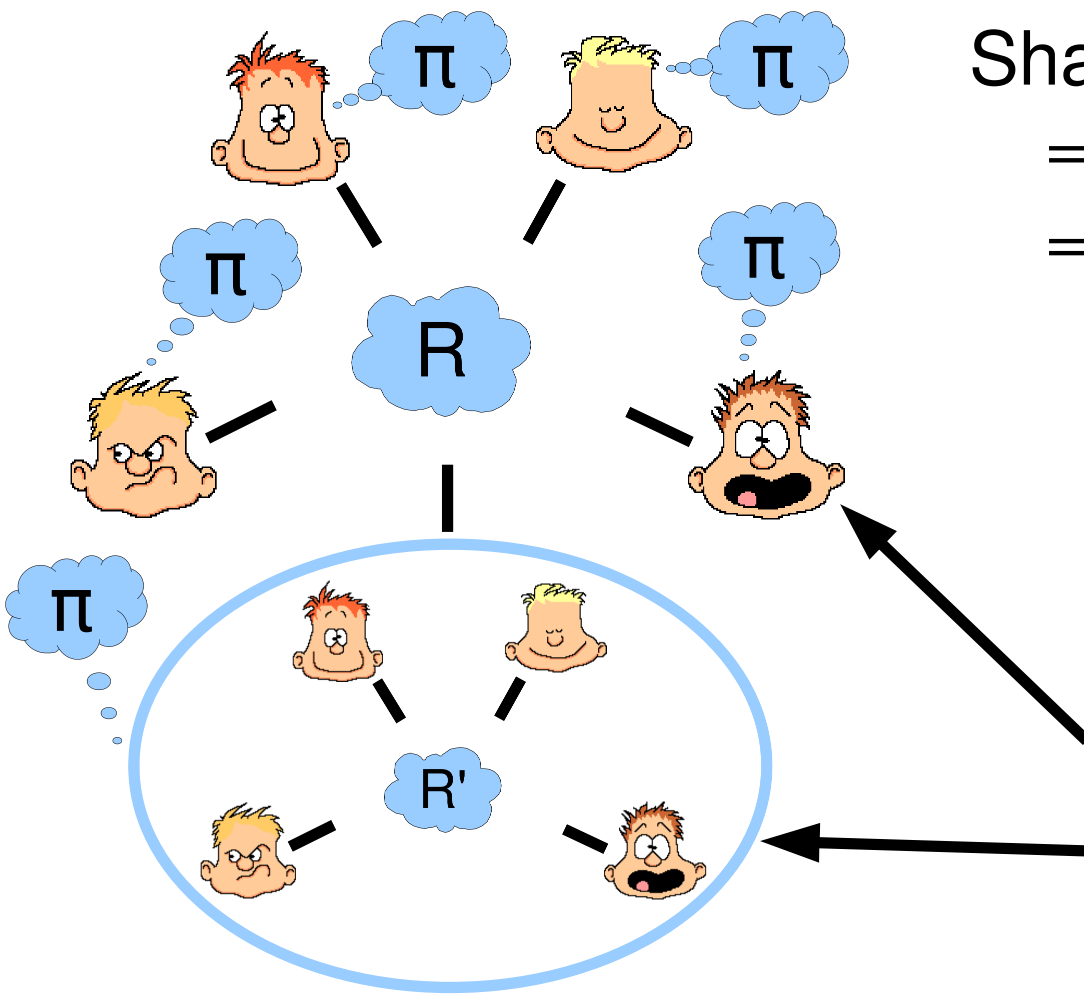
$x_i = x_i^{des} \oplus x_i^{em}$

(x_i^{des})

(x_i^{em})

>

Optimal Hybrid MPC (with BC)



Share and commit

⇒ no robustness or

⇒ no correctness for $t \geq n/2$

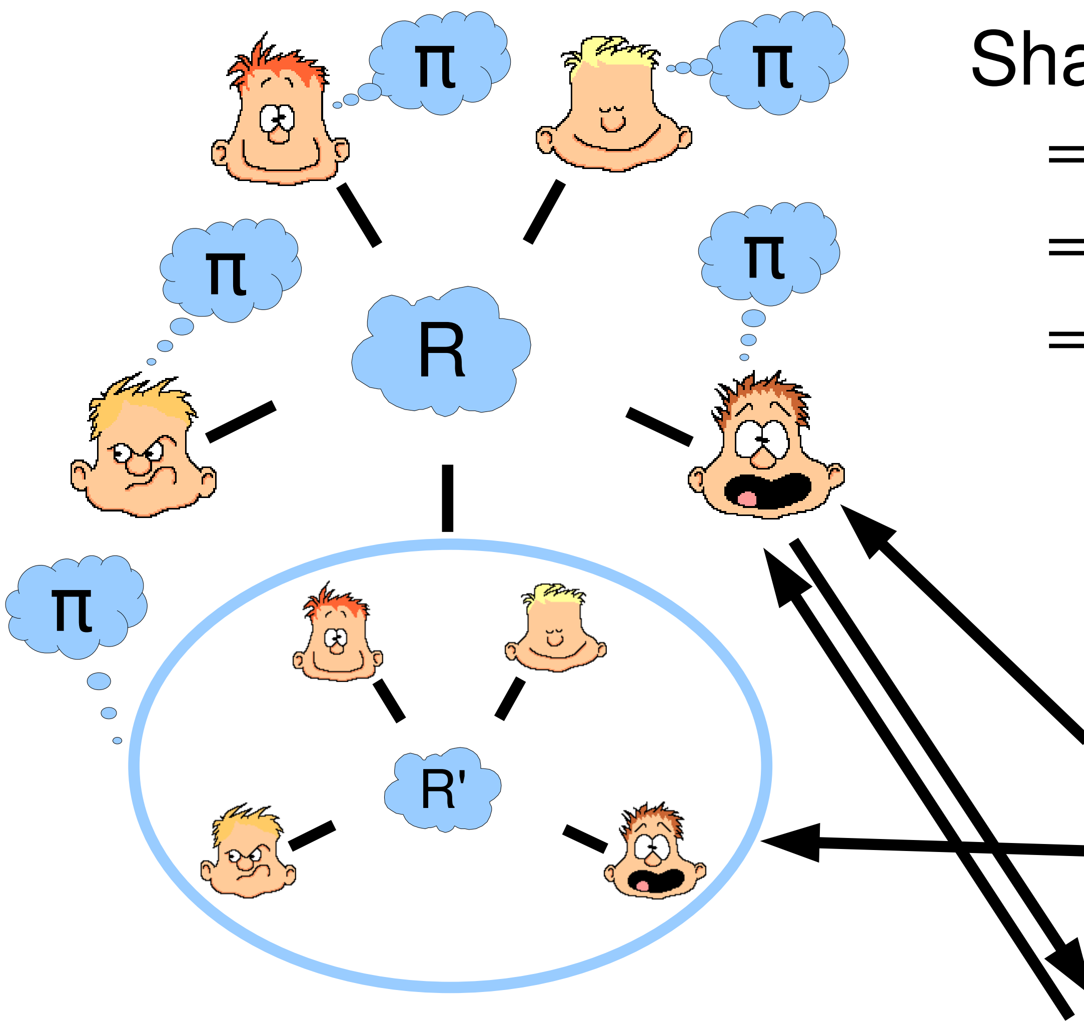
$x_i = x_i^{des} \oplus x_i^{em}$

$(c_i, o_i) = com_H(x_i^{em})$

(x_i^{des}, c_i)

(x_i^{em}, o_i)

Optimal Hybrid MPC (with BC)



Share, commit, complain

$\Rightarrow t \leq \rho$: IT full security

$\Rightarrow t < n/2$: IT fair security

$\Rightarrow t < n-\rho$: CO abort security

$x_i = x_i^{des} \oplus x_i^{em}$

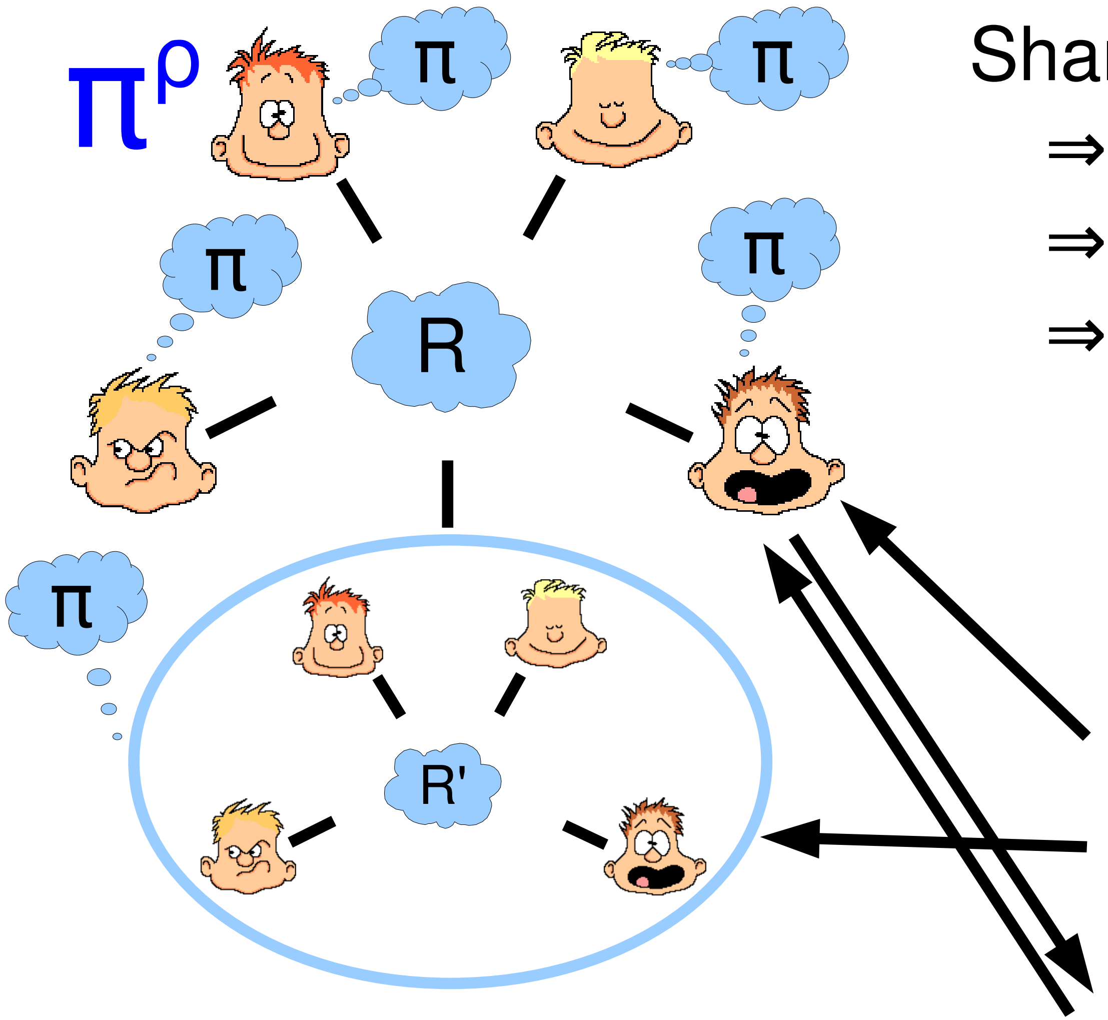
$(c_i, o_i) = com_H(x_i^{em})$

(x_i^{des}, c_i)

(x_i^{em}, o_i)

complaint? input x_i

Optimal Hybrid MPC (with BC) ✓



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Summary: Hybrid Security

- We provide optimal HMPC protocols and matching tight bounds for the setting
  - with BC

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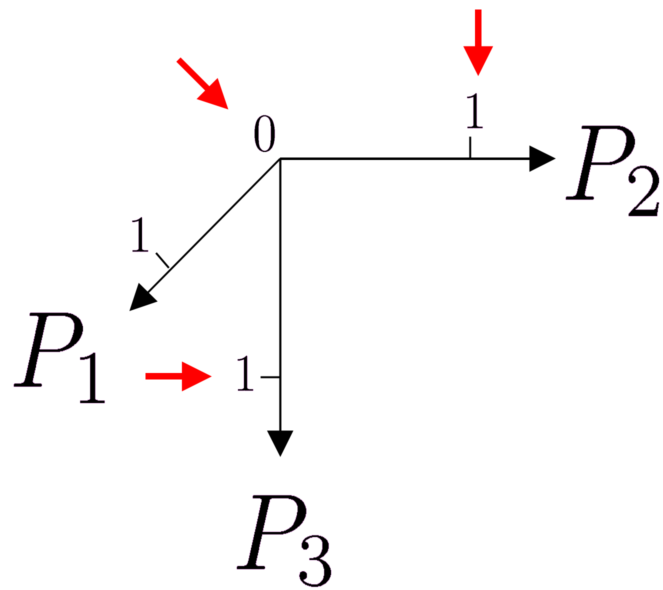
- We provide optimal HMPC protocols and matching tight bounds for the setting
  - with BC
  - without BC but with PKI
  - without BC or PKI
- We treat possibly inconsistent PKIs
- We consider signature forgery separately from other (computational) assumptions

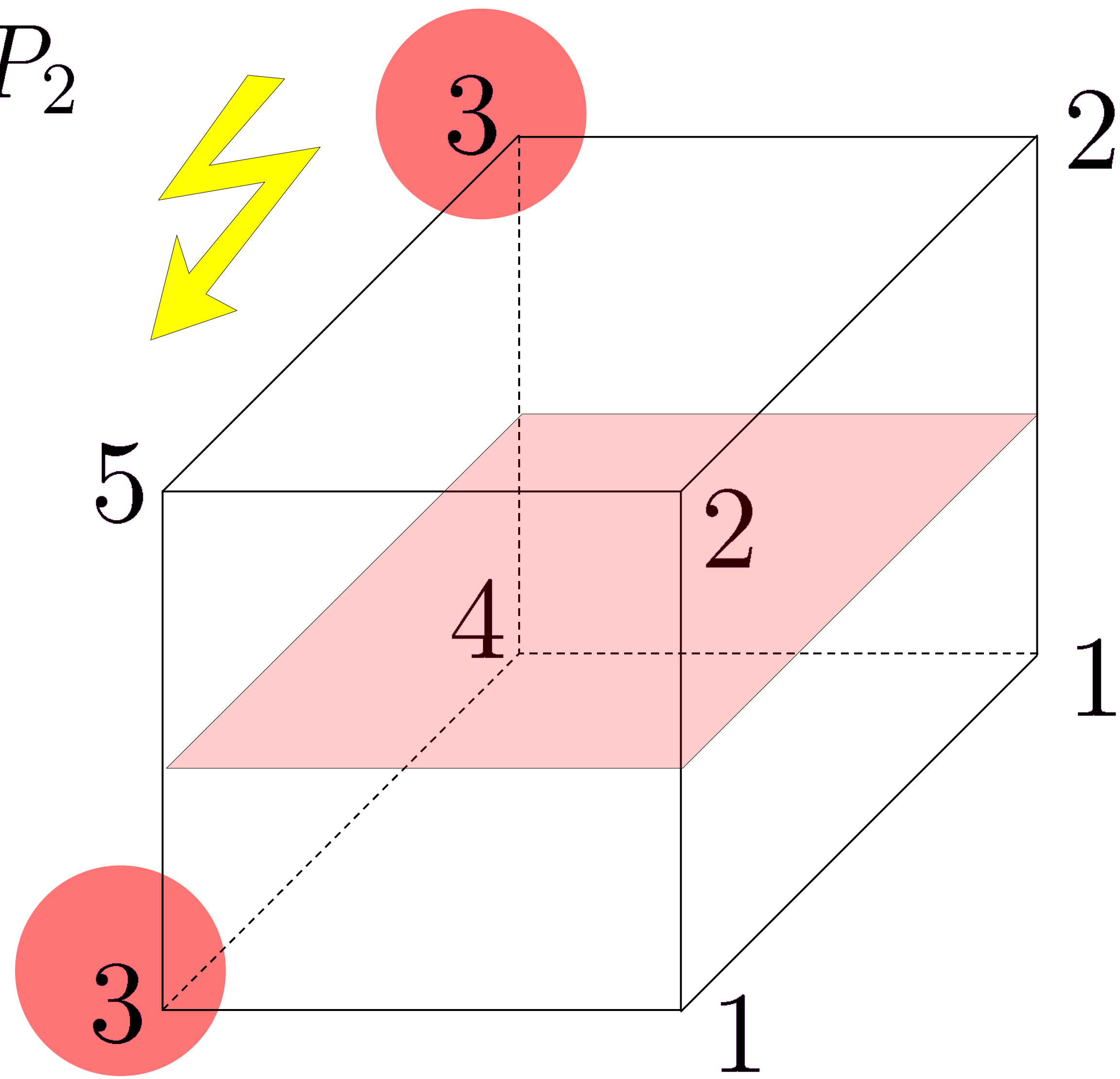
Conclusions

- Characterization of computable function classes
- Characterization of long-term security

- Optimal HMPC protocols and matching tight bounds

Passively Computable Functions F^{bc}_{pas}





Input:

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Hybrid MPC (HMPC)

- Different guarantees depending on t:
  - For $t \leq l_r$ full (robust) security
  - For $t \leq l_f$ fair security
  - For $t \leq L$ abort security
- While tolerating:
  - For $t \leq t_c$ computationally unbounded adversaries
  - For $t \leq t_\sigma$ signature forgery
  - For $t \leq t_p$ inconsistent PKIs

$\Rightarrow$ Graceful degradation

Summary: Hybrid Security

- We provide HMPC protocols for the setting
  - with BC under the bounds
    $t_c < n/2 \wedge l_r \leq l_f \leq L \wedge l_f < n/2 \wedge l_r + L < n$
  - without BC but with PKI under the bounds
    $t_c < n/2 \wedge l_r \leq l_f \leq L \wedge l_f < n/2 \wedge l_r + L < n$
    $\wedge\ 2 t_\sigma + L < n \wedge (t_p > 0 \Rightarrow t_p + 2L < n)$
  - without BC or PKI under the bounds
    $t_c < n/2 \wedge l_r \leq l_f \leq L \wedge l_f < n/2 \wedge (l_r > 0 \Rightarrow l_r + 2L < n)$
- Our bounds are tight, given $l_r \geq t_p, t_\sigma$

Limitations for HMPC with BC

- IT security for $t \leq t_c$ only if $t_c < n/2$ [Kil00]
- Fair security for $t \leq l_f$ only if $l_f < n/2$ [Cle86]
- Full security for $t \leq l_r$ and abort security for $t \leq L$ only if $l_r + L < n$ [IKLP06], [Kat07]

- Therefore:
  $t_c < n/2 \;\wedge\; l_r \leq l_f \leq L \;\wedge\; l_f < n/2 \;\wedge\; l_r + L < n \quad (1)$

Hybrid MPC without BC or PKI

- Fair security for $t \leq l_f$ only if $l_f < n/2$ [Cle86]
- IT security for $t \leq t_c$ only if $t_c < n/2$ [Kil00]
- Full security for $t \leq l_r$ and abort security for $t \leq L$ only if $l_r > 0 \Rightarrow l_r + 2L < n$ [FHHW03]
- Protocol π^ρ with the BC from [FHHW03] achieves bound $t_c < n/2 \ \wedge \ l_r \leq l_f \leq L \ \wedge \ l_f < n/2 \ \wedge \ (l_r > 0 \Rightarrow l_r + 2L < n)$ (2)
- Improves over [FHHW03] for $\rho=0$, which makes no guarantees for $t > n/2$

Limits for MPC without BC, with PKI

- Tolerate inconsistent PKI for $t \le t_p$
- Tolerate signature forgery for $t \le t_\sigma$

- We achieve the following bounds

  $t_c < n/2 \ \wedge \ l_r \le l_f \le L \ \wedge \ l_f < n/2 \ \wedge \ l_r + L < n$
  $\wedge \ 2t_\sigma + L < n \ \wedge \ (t_p > 0 \Rightarrow t_p + 2L < n) \quad (3)$

  and prove them necessary for $l_r \ge t_p, t_\sigma$

Hybrid MPC without BC, with PKI

- Protocol π^{ρ} with a hybrid BC (HBC) for bounds $2t_{\sigma}+T < n \;\wedge\; (t_p > 0 \Rightarrow t_p+2T < n)$ achieves bound (3) (where BC secure for $t \leq T$)
- For $t_p > 0$ treated in [FHW04]
- For $t_p = 0$ and $2t_{\sigma}+T < n$ we provide an HBC protocol achieving full BC
  - For $t = 0$ unconditionally
  - For $t \leq t_{\sigma}$ conditional on PKI consistency
  - For $t \leq T$ conditional on unforgeability and PKI consistency

BC with extended validity (BCEV)

- For $2t_\sigma + T < n$ and $t_p = -1$ BCEV achieves:
  - For $t \leq t_\sigma$ full broadcast
  - For $t \leq T$ validity, conditional on unforgeability

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  - For $t \le t_\sigma$ full broadcast
  - For $t \le T$ validity, conditional on unforgeability

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\}$;
3. $\texttt{if } |S_i^{x_i,0}| \ge n - T \ \wedge \ |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

   $\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi.}$ (II)

>

BCEV: Validity for t ≤ T

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\}$;

3. $\texttt{if } |S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

   $\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi}.$ (II)

>

BCEV: Validity for t ≤ T

validity:
P_s honest

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: $\mathrm{BGP}((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\};$

   $S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j} \text{valid}\};$

3. $\texttt{if } |S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

   $\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi.}$ (II)

>

BCEV: Validity for t ≤ T

validity: P_s honest

= $(m, \sigma_s(m))$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\};$

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\};$
3. **if** $|S_i^{x_i,0}| \geq n - T \;\wedge\; |S_i^{1-x_i}| = 0$ **then** $y_i := x_i$ (I)

   elsif $|S_i^0| > |S_i^1|$ **then** $y_i := 0$ **else** $y_i := 1$ **fi**. (II)

BCEV: Validity for t ≤ T

validity: P_s honest

for P_j honest = $((m,\sigma_s(m)), ?)$

= $(m,\sigma_s(m))$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\}$;
3. `if` $|S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0$ `then` $y_i := x_i$ (I)

   `elsif` $|S_i^0| > |S_i^1|$ `then` $y_i := 0$ `else` $y_i := 1$ `fi`. (II)

BCEV: Validity for t ≤ T

validity: P_s honest

for P_j honest = $((m,\sigma_s(m)), ?)$

= $(m,\sigma_s(m))$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0}\text{valid}\}$;

   $S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j}\text{valid}\}$;

3. `if` $|S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0$ `then` $y_i := x_i$ (I)

   `elsif` $|S_i^0| > |S_i^1|$ `then` $y_i := 0$ `else` $y_i := 1$ `fi`. (II)

holds always (for x_i=m)

>

BCEV: Validity for t ≤ T

validity: P_s honest

for P_j honest = $((m,\sigma_s(m)), ?)$

= $(m,\sigma_s(m))$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

$$S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0}\text{valid}\};$$

$$S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j}\text{valid}\};$$

holds for t > t_σ (and x_i=m)

3. $\texttt{if } |S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

$\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi.}$ (II)

holds always (for x_i=m)

>

BCEV: Validity for t ≤ T

validity: P_s honest

secure for $t \leq t_\sigma < n/3$

for P_j honest $= ((m,\sigma_s(m)), ?)$

$= (m,\sigma_s(m))$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

$S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

$S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\}$;

holds for $t > t_\sigma$ (and $x_i=m$)

3. **if** $|S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0$ **then** $y_i := x_i$ (I)

elsif $|S_i^0| > |S_i^1|$ **then** $y_i := 0$ **else** $y_i := 1$ **fi**. (II)

holds always (for $x_i=m$)

>

BCEV: Validity for $t \leq T$

validity: P_s honest

secure for $t \leq t_\sigma < n/3$

for P_j honest $= ((m,\sigma_S(m)), ?)$

$= (m,\sigma_S(m))$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: $\mathrm{BGP}((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j} \text{valid}\}$;
3. `if` $|S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0$ `then` $y_i := x_i$ (I)

   `elsif` $|S_i^0| > |S_i^1|$ `then` $y_i := 0$ `else` $y_i := 1$ `fi`. (II)

holds for $t > t_\sigma$ (and x_i=m)

holds always (for x_i=m)

holds for $t \leq t_\sigma$ (and m=0)

>

BCEV: Consistency for t ≤ t_σ

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\}$;

3. $\texttt{if}\ |S_i^{x_i,0}| \geq n - T \ \wedge\ |S_i^{1-x_i}| = 0\ \texttt{then}\ y_i := x_i$ (I)

   $\texttt{elsif}\ |S_i^0| > |S_i^1|\ \texttt{then}\ y_i := 0\ \texttt{else}\ y_i := 1\ \texttt{fi}.$ (II)

BCEV: Consistency for $t \leq t_\sigma$

secure for $t \leq t_\sigma < n/3$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: $\mathrm{BGP}((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j} \text{valid}\}$;
3. $\texttt{if } |S_i^{x_i,0}| \geq n - T \ \wedge\ |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

   $\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi.}$ (II)

BCEV: Consistency for $t \le t_\sigma$

secure for $t \le t_\sigma < n/3$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: $\mathrm{BGP}((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j} \text{valid}\}$;

   $S_i^v = S_j^v$

3. **if** $|S_i^{x_i,0}| \ge n - T \;\wedge\; |S_i^{1-x_i}| = 0$ **then** $y_i := x_i$ (I)

   elsif $|S_i^0| > |S_i^1|$ **then** $y_i := 0$ **else** $y_i := 1$ **fi**. (II)

>

BCEV: Consistency for t ≤ t_σ

secure for
$t \leq t_\sigma < n/3$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^{v} := \{j | v_i^{j} = v \wedge \sigma_i^{j} \text{valid}\}$;

$S_i^v = S_j^v$

3. $\texttt{if } |S_i^{x_i,0}| \geq n - T \ \wedge \ |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

   $\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi.}$ (II)

all decisions
here identical

BCEV: Consistency for $t \leq t_\sigma$

secure for $t \leq t_\sigma < n/3$

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]
2. $\forall \mathsf{P}_i$: $\mathrm{BGP}((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\}$;

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\}$;
3. $\texttt{if}\ |S_i^{x_i,0}| \geq n - T\ \wedge\ |S_i^{1-x_i}| = 0\ \texttt{then}\ y_i := x_i$ (I)

   $\texttt{elsif}\ |S_i^0| > |S_i^1|\ \texttt{then}\ y_i := 0\ \texttt{else}\ y_i := 1\ \texttt{fi}.$ (II)

$S_i^v = S_j^v$

identical S_j^v

all decisions here identical

>

BCEV: Consistency for $t \leq t_\sigma$

secure for $t \leq t_\sigma < n/3$

$j \in S_i^{v,0} \Leftrightarrow j \in S_i^v$ for P_j honest

1. P_s: multisend $(m, \sigma_s(m))$; [receive (x_i, σ_i)]

2. $\forall \mathsf{P}_i$: BGP$((x_i, \sigma_i))$; [$\forall \mathsf{P}_j$ receive $((v_i^{j,0}, \sigma_i^{j,0}), (v_i^j, \sigma_i^j))$]

   $S_i^{v,0} := \{j | v_i^{j,0} = v \wedge \sigma_i^{j,0} \text{valid}\};$

   $S_i^v := \{j | v_i^j = v \wedge \sigma_i^j \text{valid}\};$

$S_i^v = S_j^v$

identical S_j^v

3. $\texttt{if } |S_i^{x_i,0}| \geq n - T \;\wedge\; |S_i^{1-x_i}| = 0 \texttt{ then } y_i := x_i$ (I)

   $\texttt{elsif } |S_i^0| > |S_i^1| \texttt{ then } y_i := 0 \texttt{ else } y_i := 1 \texttt{ fi.}$ (II)

all decisions here identical

>

Hybrid Broadcast (HBC)

- For $2t_\sigma + T < n$ and $t_p = 0$ HBC achieves
  - For $t = 0$ full BC
  - For $t \leq t_\sigma$ full BC, conditional on PKI consistency
  - For $t \leq T$ full BC, conditional on unforgeability and PKI consistency
- Protocol idea:
  - Attempt detectable precomputation of a new PKI [FHHW03]; fall back to existing PKI
  - Run an HBC for $2t_\sigma + T < n$ and $t_p = -1$ constructed from BCEV and DS

Hybrid Broadcast (HBC) for $t_p = -1$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]
   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\ v\ :\ |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$
   and $y_i := v$; [receive S_i^j] (I)
   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]
   `If` $\exists\ v$ and a set S_i^j of valid signatures on v
   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)
   `else` $y_i := d_i$; (III)
   `fi`
   `fi`

>

HBC: Security for $t \leq t_\sigma$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\};$

4. `if` $\exists\ v\ :\ |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\ v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

HBC: Security for $t \leq t_\sigma$

BC for $t \leq t_\sigma$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]
   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v \;:\; |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$
   and $y_i := v$; [receive S_i^j] (I)
   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]
   `If` $\exists\, v$ and a set S_i^j of valid signatures on v
   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)
   `else` $y_i := d_i$; (III)
   `fi`
   `fi`

>

HBC: Security for $t \leq t_\sigma$

BC for $t \leq t_\sigma$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]
   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v : |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$
   and $y_i := v$; [receive S_i^j] (I)
   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]
   `If` $\exists\, v$ and a set S_i^j of valid signatures on v
   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)
   `else` $y_i := d_i$; (III)
   `fi`
   `fi`

holds for $t \leq t_\sigma$

>

HBC: Consistency for $t_\sigma < t \leq T$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]
   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\ v\ :\ |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$
   and $y_i := v$; [receive S_i^j] (I)
   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]
   `If` $\exists\ v$ and a set S_i^j of valid signatures on v
   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)
   `else` $y_i := d_i$; (III)
   `fi`
   `fi`

HBC: Consistency for $t_\sigma < t \leq T$

BC for $t > t_\sigma$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\ v\ :\ |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\ v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

HBC: Consistency for $t_\sigma < t \le T$

BC for t > t_σ

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\ v\ :\ |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\ v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

consistent for t > t_σ

>

HBC: Consistency for $t_\sigma < t \leq T$

BC for $t > t_\sigma$

if holds then ...

consistent for $t > t_\sigma$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v \;:\; |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\, v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

>

HBC: Consistency for $t_\sigma < t \leq T$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]

BC for $t > t_\sigma$

2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]

3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall P_j$ receive (c_i^j, σ_i^j)]

$M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;

if holds then ...

4. `if` $\exists\, v : |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

and $y_i := v$; [receive S_i^j] (I)

`else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

`If` $\exists\, v$ and a set S_i^j of valid signatures on v

and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

`else` $y_i := d_i$; (III)

`fi`

`fi`

consistent for $t > t_\sigma$

also holds for same v

>

HBC: Validity for $t_\sigma < t \leq T$

1. P_s: DS(m); [receive d_i]
2. P_s: BCEV(m); [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;

4. `if` $\exists\ v\ :\ |M_i^v| \geq n - t_\sigma$ `then` DS(M_i^v)

   and $y_i := v$; [receive S_i^j] (I)

   `else` DS$(\emptyset)$; [receive S_i^j]

   `If` $\exists\ v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

>

HBC: Validity for $t_\sigma < t \leq T$

BC for $t > t_\sigma$

guarantees validity

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v : |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\, v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

>

HBC: Validity for $t_\sigma < t \leq T$

BC for $t > t_\sigma$

guarantees validity

can only hold for v = m

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v \;:\; |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\, v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

>

HBC: Validity for $t_\sigma < t \leq T$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v : |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\, v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

BC for $t > t_\sigma$

guarantees validity

can only hold for $v = m$

can only hold for $v = m$

>

HBC: Validity for $t_\sigma < t \leq T$

1. P_s: $\mathrm{DS}(m)$; [receive d_i]
2. P_s: $\mathrm{BCEV}(m)$; [receive b_i]
3. Multisend $(b_i, \sigma_i(b_i))$; [$\forall \mathsf{P}_j$ receive (c_i^j, σ_i^j)]

   $M_i^v := \{\sigma_i^j | c_i^j = v \wedge \sigma_i^j \text{valid}\}$;
4. `if` $\exists\, v : |M_i^v| \geq n - t_\sigma$ `then` $\mathrm{DS}(M_i^v)$

   and $y_i := v$; [receive S_i^j] (I)

   `else` $\mathrm{DS}(\emptyset)$; [receive S_i^j]

   `If` $\exists\, v$ and a set S_i^j of valid signatures on v

   and $|S_i^j| \geq n - t_\sigma$ `then` $y_i := v$; (II)

   `else` $y_i := d_i$; (III)

   `fi`

   `fi`

BC for $t > t_\sigma$

guarantees validity

can only hold for $v = m$

can only hold for $v = m$

$d_i = m$

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