Categorical Semantics for Functional Reactive Programming with Temporal Recursion and Corecursion

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Functional Reactive Programming (FRP)

- programming paradigm for treating temporal aspects in a declarative fashion
- two key features:
  - time-dependent type membership
  - temporal type constructors
- Curry–Howard correspondence to temporal logic:
  - time-dependent trueness
  - temporal operators
- time:
  - linear
  - not necessarily discrete
- process consists of a **continuous part** and optionally a **terminal event**:

\[ A \triangleright \prime B: \]

- different process types with different termination guarantees:
  - nontermination possible
  - termination guaranteed
  - termination guaranteed with upper bound on termination time
Processes that deal with the present

- processes that start immediately:

\[ A \triangleright' B: \]

- processes that may terminate immediately:

\[ A \triangleright B: \]

\[ \triangleright' \text{ and } \triangleright \text{ definable in terms of } \triangleright'': \]

\[ A \triangleright' B = A \times A \triangleright'' B \quad A \triangleright B = B + A \triangleright' B \]
Abstract process categories (APCs)

- cartesian closed category $C$ with coproducts
- functors that model process type constructors:
  \[ \triangleright'' : C \times C \rightarrow C \]
- natural transformations that model FRP operations:
  - ideal monads
  - ideal comonads
  - further structure (not in this talk)
Ideal monads

- each $A \triangleright'$ is an ideal monad:

$$\mu'_B : A \triangleright' (A \triangleright B) \to A \triangleright' B$$

- concatenation of a continuous part with a follow-up process:

\[ A \triangleright' (A \triangleright B) : \quad A \triangleright' B : \]
Ideal comonads

- each $\triangleright'' B$ is an ideal comonad:

$$\delta'_A : A \triangleright'' B \to (A \triangleright' B) \triangleright'' B$$

generation of a continuous part of shorter and shorter suffixes:

- $A \triangleright'' B$:

$$A \triangleright'' B:$$

- $(A \triangleright' B) \triangleright'' B$:

$$ (A \triangleright' B) \triangleright'' B:$$
Iteration of ideal multiplication and comultiplication

- Iterated concatenation via induction:
  \[ \mu C \cdot A \triangleright' (B + C) : \]

- Iterated suffix generation via coinduction:
  \[ \nu C \cdot (A \times C) \triangleright'' B : \]
Wanted: Stronger variants of these iterations

- sequence of continuous parts may be infinite:
  \[ \nu C . A \triangleright' (B + C) : \]
  \[ A \triangleright' B : \]  

- nesting depth must be finite:
  \[ A \triangleright'' B : \]
  \[ \mu C . (A \times C) \triangleright'' B : \]
Solution: Extending the ideal monad and comonad structure

- Each $A hd'$ is a completely iterative monad:
  \[
  f : C \to A \rhd' (B + C) \\
  f^\infty : C \to A \rhd' B
  \]

- Each $\rhd'' B$ is a recursive comonad:
  \[
  f : (A \times C) \rhd'' B \to C \\
  f^* : A \rhd'' B \to C
  \]
Are these extensions reasonable?

- check whether there are nontrivial instances of APCs that have the additional structure
- concrete process categories (CPCs) are instances of APCs
- do they have the required additional structure?
Concrete process categories

- make times explicit:
  - time scale can be any totally ordered set

- express causality of operations:
  - the prefix of a result that ends at a time $t$ can only depend on the prefix of the argument that ends at $t$
  - operations expressed as families of prefix transformations, one for each $t$

- process types with simple termination guarantee cannot be modeled:
  - termination is a liveness property
  - only safety properties can be expressed, because only prefixes are considered

- the following process types can be modeled:
  - $\triangleright_{\infty}$ nontermination possible
  - $\triangleright_{t_b}$ termination at or before $t_b$ guaranteed
A constraint on time scales

- infinitely many concatenations can be problematic:

\[ \nu C . A \gtrdot^\prime (B + C) : \]

- analogous problem for suffix generation
- solution is to disallow “pathological” time scales:
  - every ascending sequence of times must be unbounded
  - Achilles catches up with the Tortoise
  - certain “interesting” time scales still allowed:

\[ \left\{ z + 1/n \mid z \in \mathbb{Z} \land n \in \mathbb{N} \setminus \{0\} \right\} \]
Compatibility with different termination (non)guarantees

- completely iterative monad:
  - $\triangleright_{\infty}$ infinitely many concatenations are no problem
  - $\triangleright_{t_b}$ only finitely many concatenations can occur, since all subprocesses terminate at or before $t_b$

- recursive comonad:
  - $\triangleright_{t_b}$ nesting depth is finite, since given process terminates
  - $\triangleright_{\infty}$ nesting depth is finite, since only finite prefixes of processes are considered
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