

Physics Cup – TalTech 2019 – Problem 1. December 30, 2018

When a body moves in liquid, the motion of the body puts the liquid into motion, too. The motion of liquid contributes to the total kinetic energy of the system, and hence, leads to an increased *effective mass* of the body. The difference of the effective mass and the actual mass of the body is referred to as the *added mass*. The added mass depends on the size and shape of the body.

Consider a certain metallic body of volume V and polarizability α along its symmetry axis x (i.e. homogeneous externally applied electric field \vec{E} induces total dipole moment $\vec{p} = \alpha\vec{E}$ on this body). Additionally, the body shape is such that if it were made from a homogeneous dielectric material and put into homogeneous electric field, the electric field inside the body would be also homogeneous. Find the added mass of this body when it starts moving translationally, parallel to the x -axis, in an incompressible initially motionless liquid of density ρ . The viscosity of the liquid is negligibly small. Express the answer in terms of V , ρ , α , and physical constants.

Hint. Initially vortex-free inviscid fluid remains free of vortices, i.e. $\oint \vec{v}d\vec{r} = 0$ for any integration contour inside the liquid, where $\vec{v} \equiv \vec{v}(\vec{r}, t)$ is the fluid velocity at position \vec{r} ; t denotes a fixed moment of time.

The hint of 16th Dec. 2018. Build an electric field with field lines matching the streamlines of liquid around the body by considering a linear combination of an homogeneous electric field, and the field created by an electrically polarized body.

The hint of 23rd Dec. 2018. In the case of the electric field from the previous hint, relate the energy of the field to the electrostatic interaction energy between the polarization charges and the external field on the one hand, and to the kinetic energy of the flow around the body on the other hand.

The hint of 30th Dec. 2018. Let us be more specific with the previous hints. Regarding the first hint, you need to show that the equations defining the velocity field and the electric field are the same. So, if the boundary conditions for the field at the surface of the body are identical in both cases, due to the uniqueness of a correctly posed problem, the solutions would be identical, too. Your task is to figure out such an electrostatic configuration for which the boundary conditions are the same as in the case of fluid flow.

Regarding the second hint, what you are suggested to do is expressing the electrostatic interaction energy as: (a) an integral of the energy density of the field ; (b) potential energy of the electric dipole (induced on the

body) in the homogeneous external electric field. (NB! be careful and do not lose the factor $\frac{1}{2}$). Since these two things must be equal, you'll obtain an expression for the integral of the energy density.

By the end of the third week, there were 372 registered participants from 54 countries; among them there were 175 high school students, and 197 university students. During the first three weeks, in total 49 solutions were submitted, out of which 16 were correct. For the university students, there is still a chance of getting the speed bonus!

Correct solutions submitted by December 30, 2018:

Name	Uni/PreUni	country	subm. time (GMT)
Yunus Emre Parmaksız	PreUni	Turkey	10 Dec. 2018, 9:36
Thomas Foster	Oxford	UK	10 Dec. 2018, 16:36
Ionel-Emilian Chiosa	PreUni	Romania	11 Dec. 2018, 18:35
Gabriel Trigo	PreUni	Brazil	12 Dec. 2018, 21:03
Oliver Lindström	PreUni	Sweden	13 Dec. 2018, 1:00
Johanes Suhardjo	HKUST	Indonesia	16 Dec. 2018, 7:44
Ivan Ridkokasha	Шевченко	Ukraine	16 Dec. 2018, 21:25
Felix Christensen	Oxford	Germany	19 Dec. 2018, 0:19
Tùng Trần	PreUni	Vietnam	19 Dec. 2018, 17:37
Maria Amcique	PreUni	France	19 Dec. 2018, 21:49
Mateusz Kapusta	PreUni	Poland	20 Dec. 2018, 18:56
Vladislav Polyakov	PreUni	Russia	21 Dec. 2018, 16:36
Farhan Husain	PreUni	Indonesia	24 Dec. 2018 7:00
Samarth Hawaldar	Indian Inst of Sci	India	25 Dec. 2018, 22:02
Mustafa Tugtekin	PreUni	Turkey	25 Dec. 2018, 22:02
Oliwier Urbański	PreUni	Poland	29 Dec. 2018, 11:31