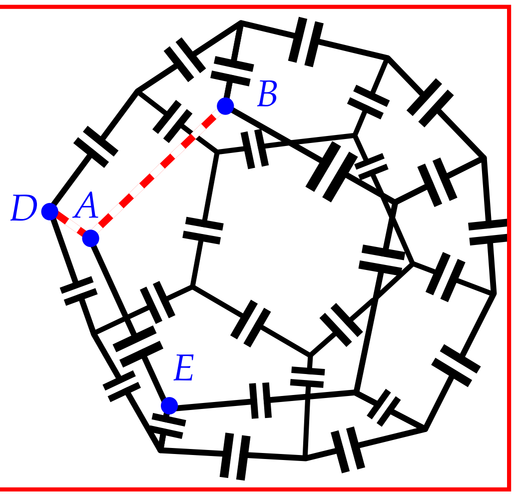
Problem 6

Edges of a dodecahedron are made of wire of negligible electrical resistance; each wire includes a capacitor of capacitance C, see figure. Let us mark a vertex A and its three neighbours B, D and E. The wire segments AB and AD are removed. What is the capacitance between the vertices B and E?



Hints after 1st week: This problem has also **a short solution which does not use brute force**. How to be sure that you have found the short solution: using the method of that short solution, it is possible to solve also a modified problem, where the dodecahedron is replaced by an infinite honeycomb lattice (two wires are cut off in the same way as for this dodecahedron).

Hints after the 2nd week: As a first step, find the resistance between B and E when the segments DA and AB (together with the respective capacitors) are still present. This can be found in the same way as the resistance r between two neighbouring nodes P and Q of an infinite square lattice of resistors R: consider the superposition of two current distributions. (i) current I is driven into the node P and driven out symmetrically at infinity; (ii) current is driven into the lattice at infinity, and out from the node Q. Due to symmetry, in both cases there is a current $I/4$ in the wire directly connecting P and Q. For the superposition, current I enters the circuit at P, and leaves from Q, and there is a current $I/4 + I/4 = I/2$ in the wire connecting P and Q, i.e. $r = R \cdot (I/2) / I = R/2$.

Hints after the 3rd week: In the case of a dodecahedron, current I, if driven into a node P, cannot be driven out at infinity because the circuit is finite. However, there is still a way to drive it out from the nodes of the dodecahedron so that (i) the current distribution remains symmetric; (ii) for a superposition of two such current distributions of opposite polarities (when adding a current distribution with I being driven out from a node Q), the external currents driven to and from all the nodes other than P and Q cancel out. Now, suppose you know the resistance r between the nodes B and E for a uncut dodecahedron: between B and E, the whole uncut dodecahedron is equivalent to a single resistor r. Next, notice that cutting out a resistor R is mathematically equivalent to adding a parallel resistance $-R$ (a mathematician doesn't care that there are no negative resistances).

Hints after the 4th week: Notice that any calculations for a certain configuration of resistances can be carried over to the equivalent configuration of capacitors. Indeed, the overall impedance $Z = 1/iC\omega$ of a system of capacitors can be found according to the rules for resistors, with resistances being substituted by the impedances $Z_k = 1/iC_k\omega$. As for the method to drive out the current which is driven into the node P, you need to drive out equal amount from every node other than P.

Intermediate conclusion after the 4th week. Correct solutions have been submitted by (ordered according to the arrival time):

1. **Lars Dehlwes (Germany)***
2. **Hrishikesh Menon (India)**
3. **Ly Nguyen (Vietnam)** (short solution was submitted later, order nubmber 14a)
4. **Dan-Cristian Andronic (Romania)**
5. **Szabo Attila (Hungary)**
6. **Jan Ondras (Slovakia)**
7. **Ng Fei Chong (Malaysia)**
8. **Tudor Ciobanu (Romania)**** (short solution was submitted later, order number 13a, i.e. between Kongas and Schmidt)
9. **Vũ Việt Linh (Vietnam)** (short solution was submitted later, order number 14b)
10. **Madhivanan Elango (United Kingdom)**

Second week begins here

11. **Nguyen Ho Nam (Vietnam)** (short solution was submitted later, order nubmber 14a)
12. **Jaemo Lim (Korea)*****
13. **Kristjan Kongas (Estonia)*****
14. **David Schmidt (Germany)*****
15. **Efim Mazhnik (Russia)*****

Third week begins here

16. **Colibaba Nicoleta (Moldova)****

Fourth week begins here

17. **Áron Dániel Kovács (Hungary)* *****
18. **Jakub Safin (Slovakia)*****
19. **Qu Xinyi (Singapore)*****

* Solution includes a typo at the very last line ** Correct version submitted at the second attempt *** Short solutions

NB! 5th week is the last one: after 7th April, solutions of Pr. 6 will no longer be accepted! On 7th April, Pr. 7 will be published. (Probs. 1.-5. will be accepted, but a penalty will apply after April 3rd).