ITT8040 Cellular Automata
Solutions to Assignment 1

Exercise 2

Let $c \in S^{\mathbb{Z}^d}$ be a configuration.

(a) Prove that there exists a sequence of finite configurations that converges to c.

It is surely possible to reason as in the proof of Proposition 4 from the notes. Let $\mathbb{Z}^d = \{\vec{r}_1, \vec{r}_2, \ldots,\}$ be an enumeration of the elements of $\mathbb{Z}^d$: fix $q \in S$, and define $c_n(\vec{r})$ as $c(\vec{r})$ if $\vec{r} = \vec{r}_i$ for some $i \leq n$, and q otherwise. Then each c_n is q-finite and $\lim_{n\to\infty} c_n = c$.

Alternatively, let $M_n = \{\vec{r} \in \mathbb{Z}^d \mid |r_i| \leq n \, \forall i \leq d\}$ the Moore neighborhood of radius n: then M_n is a d-hypercube of side $2n+1$ centered in $\vec{0}$, and for every $\vec{r} \in \mathbb{Z}^d$ there exists $n_{\vec{r}}$ such that $\vec{r} \in M_n$ for every $n \geq n_{\vec{r}}$. Let thus $c_n(\vec{r})$ be equal to $c(\vec{r})$ if $\vec{r} \in M_n$, and to q otherwise: again, c_n is q-finite and $\lim_{n\to\infty} c_n = c$.

(b) Prove that there exists a sequence of totally periodic configurations that converges to c.

The second technique from the solution of the previous part fits our current need perfectly: construct c_n so that it is $(2n+1)\vec{e}_i$-periodic for every $i \leq d$ ($\vec{e}_i$ being the i-th vector of the standard base of $\mathbb{Z}^d$, the vector whose i-th coordinate is 1 and others are 0) and coincides with c on M_n.

Observe that, in general, the period varies at each step: this is to be expected, as it is easily shown that, if $\vec{r}$ is fixed, then the limit of any converging subsequences of $\vec{r}$-periodic configurations is $\vec{r}$-periodic.