ITT8040 Cellular Automata
Assignment 4

April 3, 2013

Read pages 29–40 of Prof. Kari's notes.

1. Define the *controlled* xor by $S = \{0,1\} \times \{0,1\}$, $d = 1$, $N = \{0,1\}$, and

$$f((x_0, x_1), (y_0, y_1)) = \begin{cases} (x_0 \operatorname{xor} y_0, 1) & \text{if } x_1 = 1\,, \\ (x_0, 0) & \text{if } x_1 = 0\,. \end{cases}$$

   Let G be the global transition function of the controlled xor, and let $q = (0,0)$ be the quiescent state. Prove that G_F is surjective, but G_P is not injective.

2. Prove Proposition 21: for every *non*-surjective CA there exists a configuration with uncountably many preimages. *Hint:* Solve the exercise first in dimension $d = 1$, then in arbitrary dimension d.

3. Complete the proof of Proposition 23: for any two $c, e : \mathbb{Z} \to S$ there are pairs (k_1, k_2) of arbitrarily large integers such that $k_2 \geq k_1 + m$ and that, for every $i \in \{0, \ldots, m-2\}$, both $c(k_1 + i) = c(k_2 + i)$ and $e(k_1 + i) = e(k_2 + i)$.

4. Construct the pair graph for the elementary cellular automaton rule 174. Use the construction to find two distinct asymptotic configurations with the same image.

Soft deadline: **April 10, 2013**
Hard deadline: **April 17, 2013**