ITT8040 Cellular Automata
Solutions to Assignment 5

Exercise 1

Let $A = (S, 1, N, f)$ be a one-dimensional CA vith global function G. Consider the de Bruijn labeled graph $\Gamma = (V, E)$ of A. If a configuration c is represented by the labeling of a bi-infinite path π, then the sequence of the nodes touched by π represents, up to a fixed shift, a configuration e such that $G(e) = c$.

(a) Every non-GoE spatially periodic point has a periodic preimage

Let p be the period of c, that is, let $p > 0$ and $c(n + p) = c(n)$ for every $n \in \mathbb{Z}$. Consider a bi-infinite path π in Γ whose labeling represents c: as Γ is finite, there must be a sequence s of p consecutive edges in Γ that occurs infinitely often in π. The nodes touched between two consecutive occurrences of s define a periodic configuration which, up to a translation, is a preimage of c.

(b) If A has a fixed point then it also has a spatially periodic fixed point

Let m be the neighborhood range of the CA. Let π be a bi-infinite path in Γ whose labeling represent a fixed point c. Divide π in slices of length m: as π is infinite and the finite paths of m edges in Γ are finitely many, there must exist a slice s which is repeated infinitely often in π. The slice of π between two consecutive occurrences of s is easily seen to determine a periodic fixed point.

Exercise 2

Modify the semi-algorithm S into an algorithm S' that operates as follows:

1. First, use A to compute the value $f(x)$.
2. Store the value $f(x) + 1$ at a location n.
3. Then, reproduce the behavior of S on x, but decrease the value of n by one unit each time a step from S is performed.

4. If the answer “yes” is obtained, return “yes”.
5. If the answer “no” is obtained, return “no”.
6. If the value of n reaches zero, return “no”.

If x is a “yes” instance of P, then S returns “yes” in at most $f(x)$ steps, so S' halts and returns “yes”. If x is a “no” instance of P, then either S ultimately halts and returns “no”, or it runs for no less than $f(x) + 1$ steps, so S' halts and returns “no”.