

ITT8040 Cellular Automata – Spring 2013

Program of the course

1 Basic definitions

- Pages 1–15 of the notes.

Conway's Game of Life. Gliders, glider guns.

Configurations: dimension, set of states. Neighborhood vector. Local update rule. Global transition function.

Orbit. Space-time diagram. Special kinds of configurations: fixed-point, (temporally) periodic, eventually fixed, eventually (temporally) periodic. Special neighborhoods: von-Neumann, Moore. Support, quiescent state, quiescent configuration.

Elementary CA. Wolfram's classification.

Translations. Configurations: finite, spatially periodic, totally periodic. Compactness principle.

2 Basic theorems

- Pages 16–29 of the notes.

Injectivity. Surjectivity. Reversibility.

Balancedness. Garden-of-Eden configurations. Orphan patterns.

Balancedness theorem.

Pre-injectivity. Garden-of-Eden theorem.

3 One-dimensional cellular automata

- Pages 29–41 of the notes.

One dimensional case. Theorems valid in dimension 1 but not higher. Examples.

De Bruijn graph of order m over an alphabet S . Labeled de Bruijn graph of a one-dimensional CA. Pair graph of a one-dimensional CA. Criteria for injectivity and surjectivity of a one-dimensional CA.

4 Two-dimensional CA and tilings

- Page 41–49 of the notes.

Tilings. Tile sets. Valid tilings. Wang tile sets. Periodic tilings. Aperiodic tile sets. NW-deterministic tile sets. The plane filling property.
Cellular automata from tile sets. Counterexamples in dimension 2.

5 Algorithmic questions

- Page 50–75 of the notes.

Semi-algorithms. Decidability. Semi-decidability. Undecidable problems. Many-to-one reduction.

Turing machines. Turing's halting problem.

Undecidable problems on tilings: tiling problem, tiling problem with seed tile, finite tiling problem, periodic tiling problem, NW-deterministic tiling problem.

Nilpotent CA. Undecidability of CA nilpotency.

Undecidability of reversibility of 2D CA.

Undecidability of surjectivity of 2D CA.

R.e.-completeness. R.e.-complete problems.

6 Reversible CA

- Page 75–87 of the notes, plus slides.

Lattice gas cellular automata. HPP. FHP.

Partitioned CA. Representation of arbitrary CA by reversible partitioned CA.

Block CA. Margolus neighborhood. HPP with the Margolus neighborhood. Fredkin's billiard ball model. BBM with the Margolus neighborhood.

Second-order CA.

7 Conserved quantities

- Pages 87–93 of the notes.

Definitions. Equivalence of the formulations for finite and for periodic configurations. Hattori-Takesue conditions.