Mathematics for Computer Science
Self-evaluation exercises for Lecture 1

Silvio Capobianco

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Exercise 1.1 (cf. Problem 1.9 in the textbook)

Let $|x| = \max(x, -x)$ be the *absolute value* of the real number x. Prove by cases the *triangle inequality*: for every two real numbers x and y,

$$|x + y| \leq |x| + |y| \ .$$

Exercise 1.2

We have seen in classroom a proof of the implication:

If $1 = -1$, then $2 = 1$.

Modify the argument to obtain a proof of the following implication:

If $2 + 2 = 5$, then I am the Pope.

(There are proofs available in the literature and on the Web, but it is good to try by oneself first.)

Exercise 1.3 (from the classroom test of 03.10.2018)

1. Prove that $\log_{20} 50$ is irrational.

2. Let $a > 1$ and $b > 1$ be integers. Can $\log_a b$ be rational if b is not a power of a?

Exercise 1.4 (cf. Problem 1.19)

An integer m is a *divisor* of an integer n if there exists an integer k such that $m \cdot k = n$. Note that, with this definition, every integer is a divisor of 0.

Let $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_d x^d$ be a polynomial of degree $d \geq 1$ with integer coefficients. The *rational root theorem* says that, if for two relatively prime integers m, n the value m/n is a root of $p(x)$ (that is, $p(m/n) = 0$) then m is a divisor of the *constant term* a_0 and n is a divisor of the *leading coefficient* a_d.

1. Prove the rational root theorem.

2. Use the rational root theorem to prove that, if the integer k is not the rth power of some other integer, then the rth root of k is irrational.

   Note: you *do not* need to have solved point 1 before you solve point 2, but you *must* use it.