Mathematics for Computer Science
Self-evaluation exercises for Chapter 3

Silvio Capobianco

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Exercise 3.1 (cf. Problem 3.4)

Describe a simple procedure which, given a positive integer argument, n, produces a width n array of truth-values whose rows would be all the possible truth-value assignments for n propositional variables. For example, for $n = 2$, the array would be:

T	**T**
T	**F**
F	**T**
F	**F**

Your description can be in English, or a simple program in some familiar language such as Python or Java. If you do write a program, be sure to include some sample output.

Exercise 3.2 (from the classroom test of 3 October 2018)

Find a disjunctive normal form for the following formula:

$$(P \textbf{ or } Q) \textbf{ implies } \textbf{not}(R \textbf{ and } P)$$

Use either a truth table, or logical equivalences.

Exercise 3.3 (cf. Problem 1.16(c)-(d))

1. Let P be a propositional formula. Explain why P is valid if and only if $\textbf{not}(P)$ is *not* satisfiable.

2. A finite set of propositional formulas $X = \{P_1, \ldots, P_n\}$ is *consistent* if there exists an assignment of truth values to all the variables which appear in any formulas in which all propositions are true. For example, the following set is consistent:

$$\{P \textbf{ and not}(Q), Q \textbf{ or } R\},$$

but the following one is not:

$$\{A \textbf{ and not}(A)\}.$$

Construct a formula S such that S is valid if and only if X is *not* consistent.

Exercise 3.4 (cf. Problem 3.18(c))

We have seen during the classroom exercises that every propositional formula can be rewritten as an equivalent formula where only the connectives **or** and **not**() appear. Consider now the operator **nand** defined by:

$$A \textbf{ nand } B ::= \textbf{not}(A \textbf{ and } B).$$

Prove that every propositional formula can be rewritten as an equivalent formula where only the connective **nand** appears.

Exercise 3.5 (cf. Problem 3.28)

Express each of the following statements using quantifiers, logical connectives, and/or the following predicates:

- $P(x)$ ::= 'x is a monkey"
- $Q(x)$ ::= 'x is a 6.042 TA"
- $R(x)$ ::= 'x comes from the 23rd century"
- $S(x)$ ::= 'x likes to eat pizza"

where x ranges over all living things.

(a) No monkey likes to eat pizza.

(b) Nobody from the 23rd century dislikes eating pizza.

(c) All 6.042 TAs are monkeys.

(d) No 6.042 TA comes from the 23rd century.

(e) Does part (d) follows from parts (a), (b) and (c)? If so, give a proof. If not, give a counterexample.

(f) Translate into English: $\forall x\,.\,(R(x)\ \textbf{or}\ S(x)\ \textbf{implies}\ Q(x))$

(g) Translate into English:

$$\exists x\,.\,(R(x)\ \textbf{and}\ \textbf{not}(Q)(x))\ \textbf{implies}\ \forall x\,.\,(P(x)\ \textbf{implies}\ S(x))$$

Exercise 3.6 (from the classroom test of 3 October 2018)

Find a counter-model for the following predicate formula:

$$(\exists x\,.\,\forall y\,.\,(P(x)\ \textbf{implies}\ Q(y)))\ \ \textbf{implies}\ \ (\forall x\,.\,(P(x)\ \textbf{implies}\ \exists y\,.\,Q(y)))\ .$$