First-Class Signals for Functional Reactive Programming

Wolfgang Jeltsch

TTÜ Küberneetika Instituut

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Overview

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Memoization

Start time consistency

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Functional Reactive Programming

- the ideal reactive system:
  - continuous change
  - immediate, atomic reactions on events
- not reflected by imperative implementations:
  - discretization visible
  - inconsistent intermediate states visible
- programmer confronted with technical details:
  - polling loops
  - event handlers
- goal of functional programming:

  problem description instead of execution plan
- Functional Reactive Programming (FRP):

  applying this principle to reactive systems

Implementations

- two ways of implementing FRP:

pull-based	system state is repeatedly recomputed
push-based	state changes are triggered by events

- many Haskell EDSLs:
  - pull-based:
    - Fran
    - Yampa

    etc.
  - push-based:
    - FranTk
    - Reactive
    - Grapefruit

    etc.
- EDSLs for other programming languages (all push-based):

Java	Frappé
Scheme	FrTime
JavaScript	Flapjax

Grapefruit

- originally geared towards GUI programming
- push-based, because change is rare in GUIs
- problem with existing push-based implementations:
  - no first-class descriptions of temporal behavior
  - performance problems
- a new implementation for solving these issues

Overview

Signals

- the heart of FRP
- describe temporal behavior
- three flavors:

discrete: values associated with discrete times:

$$[\![\mathit{DSignal}]\!]\ \alpha \approx [(\mathit{Time}, \alpha)]$$

continuous: arbitrary time-varying values:

$$[\![\mathit{CSignal}]\!]\ \alpha \approx \mathit{Time} \to \alpha$$

segmented: step functions over time:

$$[\![\mathit{SSignal}]\!]\ \alpha = (\alpha, [\![\mathit{DSignal}]\!]\ \alpha)$$

Examples of signals

discrete incoming network packets:

$$\mathit{DSignal}\ \mathit{Packet}$$

continuous time since the program has started:

$$\mathit{CSignal}\ \mathit{DiffTime}$$

segmented amount of network traffic:

$$\mathit{SSignal}\ \mathit{Integer}$$

Signal combinators

- functions for constructing signals
- some examples:

$$\begin{aligned} \mathit{union} &:: \mathit{DSignal}\ \alpha \to \mathit{DSignal}\ \alpha \to \mathit{DSignal}\ \alpha \\ \mathit{filter} &:: (\alpha \to \mathit{Bool}) \to \mathit{DSignal}\ \alpha \to \mathit{DSignal}\ \alpha \\ \mathit{scanl} &:: (\beta \to \alpha \to \beta) \to \beta \to \mathit{DSignal}\ \alpha \\ &\to \mathit{SSignal}\ \beta \end{aligned}$$

- application of these combinators:

$$\begin{aligned} &\ddot{p} :: \mathit{DSignal}\ \mathit{Packet} \\ &\ddot{p} = \mathit{union}\ \ddot{p}_{\mathit{In}}\ \ddot{p}_{\mathit{Out}} \end{aligned}$$

$$\begin{aligned} &\ddot{p}_{\mathit{TCP}} :: \mathit{DSignal}\ \mathit{Packet} \\ &\ddot{p}_{\mathit{TCP}} = \mathit{filter}\ \mathit{isTCPPacket}\ \ddot{p} \end{aligned}$$

$$\begin{aligned} &\bar{v} :: \mathit{SSignal}\ \mathit{Integer} \\ &\bar{v} = \mathit{scanl}\ (\lambda v\ p \to v + \mathit{size}\ p)\ 0\ \ddot{p} \end{aligned}$$

Switching

- class *Signal* of all signal types
- switching combinator:

  $$\mathit{switch} :: (\mathit{Signal}\ \sigma) \Rightarrow \mathit{SSignal}\ (\sigma\ \alpha) \rightarrow \sigma\ \alpha$$
- possible application:
  - two segmented signals that represent amount of incoming and outgoing traffic:

    $$\bar{v}_{\mathit{In}}, \bar{v}_{\mathit{Out}} :: \mathit{SSignal}\ \mathit{Integer}$$
  - segmented signal that toggles between these two, depending on user selection:

    $$\bar{\bar{v}} :: \mathit{SSignal}\ (\mathit{SSignal}\ \mathit{Integer})$$
  - switching creates a signal that always gives the respective amount of traffic:

    $$\bar{v}_{\mathit{Sel}} :: \mathit{SSignal}\ \mathit{Integer}$$
    $$\bar{v}_{\mathit{Sel}} = \mathit{switch}\ \bar{\bar{v}}$$
  - $\bar{v}_{\mathit{Sel}}$ used as the input of a display component

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A straightforward push-based implementation

- updates shall be event-driven
- signal consumers register event handlers
- discrete signal is registration action
  (which yields unregistration action):

  $$\textbf{type}\ \mathit{Handler}\ \alpha = \alpha \to \mathit{IO}\ ()$$

  $$\textbf{type}\ \mathit{DSignal}\ \alpha = \mathit{Handler}\ \alpha \to \mathit{IO}\ (\mathit{IO}\ ())$$

- *SSignal* implementation directly mirrors the semantics:

  $$\textbf{type}\ \mathit{SSignal}\ \alpha = (\alpha, \mathit{DSignal}\ \alpha)$$

Implementation of *scanl*

$$
\begin{array}{l}
\mathit{scanl} :: (\beta \rightarrow \alpha \rightarrow \beta) \rightarrow \beta \rightarrow \mathit{DSignal}\ \alpha \rightarrow \mathit{SSignal}\ \beta \\
\mathit{scanl}\ f\ y_0\ \ddot{x} = (y_0, \ddot{y})\ \textbf{where} \\
\quad \ddot{y} = \lambda h \rightarrow \textbf{do} \\
\qquad\qquad\qquad\qquad \vec{y} \leftarrow \mathit{newIORef}\ y_0 \\
\qquad\qquad\qquad\qquad \ddot{x}\ (\lambda x \rightarrow \textbf{do} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad y \leftarrow \mathit{readIORef}\ \vec{y} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \textbf{let} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad y' = f\ y\ x \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \mathit{writeIORef}\ \vec{y}\ y' \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad h\ y')
\end{array}
$$

Generators, not signals

- registration actions executed once per consumer
- when using *scanl*, every consumer
  - creates a mutable variable, initialized at consumption time
  - registers a handler that updates this variable
- two problems:
  1. duplication of computations
  2. signal values depending on consumption time
- intuition:
  - values of signal types are in fact generators
  - generator yields a new signal when consumed
  - signals are not first-class anymore

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Using native memoization

- Haskell caches computed parts of a data structure if a variable is bound to the structure
- problem:

  values of *DSignal* do not contain event values
- changing the data structure:

  type *DSignal* $\alpha = [(\mathit{Time}, \alpha)]$
- event streams must be interleaved when computing signal unions:

$$\begin{aligned} \mathit{union}\ ((t_1, x_1) : \ddot{x}_1)\ ((t_2, x_2) : \ddot{x}_2)\ &|\ t_1 < t_2 = \cdots \\ &|\ t_1 \equiv t_2 = \cdots \\ &|\ t_1 > t_2 = \cdots \end{aligned}$$

- problem:

  comparison of occurrence times must succeed when the first event occurs
- our solution:

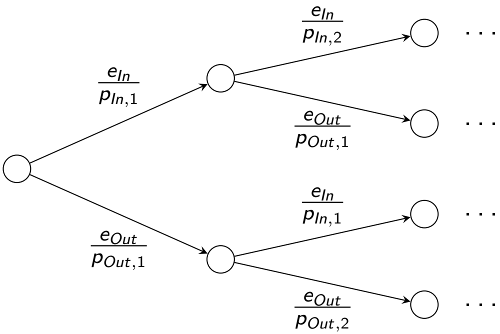
  delegate event ordering to the consumers

Representing discrete signals by vistas

- vista covers every possible event stream interleaving
- future behavior depends on which external event source fires next:

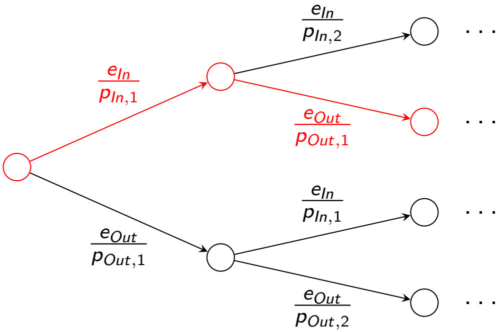
$$\begin{aligned} &\textbf{type}\ \mathit{Vista}\ \ \alpha = \mathit{Map}\ \mathit{EventSrc}\ (\mathit{Variant}\ \alpha) \\ &\textbf{type}\ \mathit{Variant}\ \alpha = (\alpha, \mathit{Vista}\ \alpha) \end{aligned}$$

- vista for *union* $\ddot{p}_{In}$ $\ddot{p}_{Out}$:



Consuming vistas

- consumer knows about the order in which event sources fire
- evaluates only the relevant path:



Implementation of combinators

- functional representation of discrete signals leads to functional implementations of combinators
- implementation of *scanl*:

$$\begin{aligned} &\mathit{scanl} :: (\beta \rightarrow \alpha \rightarrow \beta) \rightarrow \beta \rightarrow \mathit{DSignal}\ \alpha \\ &\qquad \rightarrow \mathit{SSignal}\ \beta \\ &\mathit{scanl}\ f\ y_0\ \ddot{x} = (y_0, a\ y_0\ \ddot{x})\ \textbf{where} \\ &\quad a\ y = \mathit{fmap}\ (\lambda(x, \ddot{x}) \rightarrow \textbf{let} \\ &\qquad\qquad\qquad\qquad\qquad y' = f\ y\ x \\ &\qquad\qquad\qquad\qquad\quad \textbf{in}\ (y', a\ y'\ \ddot{x})) \end{aligned}$$

- problem with *filter*:

  removing nodes would destroy structure of the vista
- solution:

  make event values optional
- modified *Variant* type:

$$\textbf{type}\ \mathit{Variant}\ \alpha = (\mathit{Maybe}\ \alpha, \mathit{DSignal}\ \alpha)$$

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Fixing start times

- technique inspired by Haskell's *ST* monad
- signal types get an extra (phantom) type parameter that represents signal start times
- signal combinators enforce start time equality:

$$\begin{aligned} \mathit{union} &:: \mathit{DSignal}\ t_0\ \alpha \to \mathit{DSignal}\ t_0\ \alpha \\ &\to \mathit{DSignal}\ t_0\ \alpha \\ \mathit{scanl} &:: (\beta \to \alpha \to \beta) \to \beta \to \mathit{DSignal}\ t_0\ \alpha \\ &\to \mathit{SSignal}\ t_0\ \beta \end{aligned}$$

- actions for producing and consuming signals have a parameter representing execution time:

$$\textbf{newtype}\ \mathit{Reactive}\ t_0\ \alpha = \mathit{Reactive}\ (\mathit{IO}\ \alpha)$$

- signal production and consumption enforce start time equality
- conversion to *IO* uses universal quantification:

$$\mathit{toIO} :: (\forall t_0 . \mathit{Reactive}\ t_0\ \alpha) \to \mathit{IO}\ \alpha$$

Safe switching

- safe switching combinator:

$$\mathit{switch} :: (\mathit{Signal}\ \sigma) \Rightarrow \mathit{SSignal}\ t_0\ (\forall t. \sigma\ t\ \alpha) \to \sigma\ t_0\ \alpha$$

- switches only to signals that don't depend on external events:
  - empty discrete signal
  - constant continuous signals
  - constant segmented signals
- useless
- idea:

  switching between signal functions
  instead of signals

Signal functions to the rescue

- functions on signals with identical start time:

$$\mathit{SigFun}\ t_0\ (\sigma_1\ \text{`}\mathit{Of}\text{`}\ \alpha_1 \mapsto \cdots \mapsto \sigma_n\ \text{`}\mathit{Of}\text{`}\ \alpha_n \mapsto \sigma\ \text{`}\mathit{Of}\text{`}\ \alpha)$$

- empty data types for type indices:

$$\textbf{data}\ \varphi \mapsto \varphi'$$

$$\textbf{data}\ \sigma\ \text{`}\mathit{Of}\text{`}\ \alpha$$

- *SigFun* defined as a GADT:

$$\begin{aligned}
&\textbf{data}\ \mathit{SigFun}\ t_0\ \varphi\ \textbf{where}\\
&\quad \mathit{SigFun}_0 \quad :: (\mathit{Signal}\ \sigma)\\
&\quad\quad \Rightarrow \sigma\ t_0\ \alpha\\
&\quad\quad \rightarrow \mathit{SigFun}\ t_0\ (\sigma\ \text{`}\mathit{Of}\text{`}\ \alpha)\\
&\quad \mathit{SigFun}_{succ} :: (\mathit{Signal}\ \sigma)\\
&\quad\quad \Rightarrow (\sigma\ t_0\ \alpha \rightarrow \mathit{SigFun}\ t_0\ \varphi')\\
&\quad\quad \rightarrow \mathit{SigFun}\ t_0\ (\sigma\ \text{`}\mathit{Of}\text{`}\ \alpha \mapsto \varphi')
\end{aligned}$$

Switching between signal functions

- type of the switching combinator:

$$\begin{aligned} \mathit{switch} &:: \mathit{SSignal}\ t_0\ (\forall t.\mathit{SigFun}\ t\ \varphi) \\ &\quad \to \mathit{SigFun}\ t_0\ \varphi \end{aligned}$$

- how the combinator works (conceptionally):
  - arguments of the result function are pruned to fit the segments of the argument signal (ageing)
  - each function from the argument signal is applied to its corresponding slices
  - resulting segments are concatenated

The traffic volume example again

- type of binary signal functions over a single signal type:

$$\textbf{type}\ \mathit{BinSigFun}\ t_0\ \sigma\ \alpha = \mathit{SigFun}\ t_0\ (\sigma\ \text{`}\mathit{Of}\text{`}\ \alpha \mapsto \sigma\ \text{`}\mathit{Of}\text{`}\ \alpha \mapsto \sigma\ \text{`}\mathit{Of}\text{`}\ \alpha)$$

- projection functions:

$$\begin{aligned}
&\pi_1, \pi_2 :: \mathit{BinSigFun}\ t_0\ \sigma\ \alpha \\
&\pi_1 = \mathit{SigFun}_{succ}\ \$\ \lambda s_1 \rightarrow \\
&\qquad \mathit{SigFun}_{succ}\ \$\ \lambda\_ \rightarrow \mathit{SigFun}_0\ s_1 \\
&\pi_2 = \mathit{SigFun}_{succ}\ \$\ \lambda\_ \rightarrow \\
&\qquad \mathit{SigFun}_{succ}\ \$\ \lambda s_2 \rightarrow \mathit{SigFun}_0\ s_2
\end{aligned}$$

- segmented signal that toggles between these functions:

$$\bar{f} :: \mathit{SSignal}\ t_0\ (\forall t.\mathit{BinSigFun}\ t\ \sigma\ \alpha)$$

- switching yields time-varying projection:

$$\begin{aligned}
&f :: \mathit{BinSigFun}\ t_0\ \sigma\ \alpha \\
&f = \mathit{switch}\ \bar{f}
\end{aligned}$$

- unpacking and applying to $\bar{v}_{In}$ and $\bar{v}_{Out}$ yields $\bar{v}_{Sel}$

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